Effective Theory of a Light Dilaton

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We consider scenarios where strong conformal dynamics constitutes the ultraviolet completion of the physics that drives electroweak symmetry breaking. We show that in theories where the operator responsible for the breaking of conformal symmetry is close to marginal at the breaking scale, the dilaton mass can naturally lie below the scale of the strong dynamics. However, in general this condition is not satisfied in the scenarios of interest for electroweak symmetry breaking, and so the presence of a light dilaton in these theories is associated with mild tuning. We construct the effective theory of the light dilaton in this framework, and determine the form of its couplings to Standard Model states. We show that corrections to the form of the dilaton interactions arising from conformal symmetry violating effects are suppressed by the square of the ratio of the dilaton mass to the strong coupling scale, and are under good theoretical control. These corrections are generally subleading, except in the case of dilaton couplings to marginal operators, when symmetry violating effects can sometimes dominate. We investigate the phenomenological implications of these results for models of technicolor, and for models of the Higgs as a pseudo-Nambu-Goldstone boson, that involve strong conformal dynamics in the ultraviolet.

I. INTRODUCTION

Although the Standard Model (SM) has experienced more than 30 years of experimental successes, the nature of the dynamics that underlies the electroweak phase transition has remained a mystery. Although several promising theories have been put forward, each faces its own challenges and conclusive experimental evidence that could settle the issue one way or the other has been lacking. The recent discovery of a new particle with mass close to 125 GeV and properties similar to that of the SM Higgs boson [1, 2] will help resolve this issue. The simplest possibility, and the one favored by precision electroweak tests, is that the new particle is indeed the SM Higgs. In this case the remaining challenge is to explain the stability of the weak scale under radiative corrections, the 'hierarchy problem'. If, however, electroweak symmetry is broken by strong dynamics, as in technicolor models [3, 4], for a review see [5], there is no immediate understanding of the origin of the new particle, or an explanation of why its properties are similar to that of the SM Higgs. The simplest technicolor models remain disfavored by precision tests [6–8], and there is no immediate understanding of the absence of the new contributions to flavor changing neutral currents that are expected to be generated by the mechanism that gives the SM quarks and leptons their masses.

There have long been good reasons to think that strong conformal dynamics may play a role in electroweak symmetry breaking, irrespective of the existence of a light Higgs. In theories of technicolor, if the strong dynamics that breaks electroweak symmetry is conformal in the ultraviolet, the operators that give rise to the fermion masses can have a large anomalous dimension. This framework is used in conformal technicolor models [9] to generate a natural separation of the flavor scale from the electroweak scale, allowing the experimental limits on flavor violation to be satisfied. (This approach to the flavor problem was first proposed in the context of walking technicolor [10–13], which is closely related to conformal technicolor.) In theories with a light Higgs, one class of promising solutions to the hierarchy problem are those where the SM Higgs emerges as the pseudo-Nambu-Goldstone boson of a global symmetry that is broken by strong dynamics [14–16]. This class of theories includes little Higgs models [17–19], and twin Higgs models [20, 21]. If the ultraviolet physics involves strong conformal dynamics, the flavor scale can again be separated from the electroweak scale, allowing new contributions to flavor violating processes to be small enough to satisfy the existing constraints.

In theories where an exact conformal symmetry is spontaneously broken, the low energy effective theory below the breaking scale contains a massless scalar, the dilaton, which may be thought of as the Nambu-Goldstone boson (NGB) associated with the breaking of conformal symmetry [22–25]. The form of the dilaton couplings is fixed by the requirement that conformal symmetry be realized nonlinearly, and so this framework is extremely predictive. Several authors have studied the couplings of a light dilaton in the context of theories of electroweak symmetry breaking [26–28]. Remarkably, the interactions of a dilaton with the SM fields are very similar to those of the SM Higgs [27]. This can be traced to the fact that at the classical level the SM has an approximate conformal symmetry which is spontaneously broken by the VEV of the Higgs, so that the Higgs can be understood as a dilaton in this limit. However, in the class of theories of interest for electroweak symmetry breaking, conformal symmetry is expected to be explicitly violated by operators that grow in the infrared to become strong at the breaking scale. Therefore, in general, there is no reason to expect a light dilaton in the low energy effective theory.

In this paper we consider scenarios where strong conformal dynamics constitutes the ultraviolet completion of

the physics that drives electroweak symmetry breaking. Following the framework outlined in [29], we show that in theories where the operator responsible for breaking conformal symmetry is marginal at the breaking scale, the dilaton mass can naturally lie below the scale of the strong dynamics.* However, in general this condition is not satisfied by the theories of interest for electroweak symmetry breaking, and so the presence of a light dilaton in these theories is associated with mild tuning. We construct the effective theory of the light dilaton in this framework, and determine the form of its couplings to SM states. We show that corrections to the form of the dilaton interactions arising from conformal symmetry violating effects are suppressed by the square of the ratio of the dilaton mass to the strong coupling scale, and are under good theoretical control. These corrections are subleading, except in the case of dilaton couplings to marginal operators, when they can sometimes dominate.

These results have important implications for our understanding of electroweak symmetry breaking. One possibility is that the new particle that has been observed close to 125 GeV is not the SM Higgs, but instead a dilaton that emerges from a strongly interacting conformal sector that breaks electroweak symmetry dynamically. In fact, several papers that interpret the 125 GeV resonance as a dilaton have already appeared in the literature [31–34]. In such a scenario an understanding of the general form of the dilaton couplings, including conformal symmetry violating effects, is crucial to distinguishing it from the SM Higgs. Another possibility is that the new particle which has been observed is indeed the SM Higgs, which arises as the pseudo-Nambu-Goldstone boson (pNGB) of an approximate global symmetry that is broken by strong conformal dynamics. Our analysis shows that in such a scenario, there may be an additional light scalar in the low energy effective theory beyond the SM Higgs whose couplings to the SM fields can be predicted.

The AdS/CFT correspondence [35–38] can be used to relate Randall-Sundrum models in warped extra dimensions [39] to strongly coupled conformal field theories in the large N limit. In this way, extra dimensional realizations of technicolor [40] and of the Higgs as a pNGB [41] have been obtained. In the correspondence, the radion in the Randall-Sundrum model is identified with the dilaton [26]. Radion stabilization using the Goldberger-Wise mechanism [42] can be understood as a stable minimum for the dilaton potential being generated by effects which explicitly violate conformal symmetry [26]. Several authors have studied the couplings of the radion in Randall-Sundrum models, both in the case when the SM fields are localized to a brane [42–44] and in the case when they are in the bulk [45, 46]. We find excellent agreement between these results and ours in the regime when the large N approximation is valid.

II. EFFECTIVE THEORY OF A DILATON

In this section we construct the effective theory for the dilaton, incorporating conformal symmetry violating effects, and show that if the operator that breaks conformal symmetry is marginal at the breaking scale, the dilaton can naturally be light.

The fifteen parameter conformal group extends the ten parameter Poincare group to include scale transformations and special conformal transformations. While it has long been conjectured that any Poincare invariant, unitary theory that realizes scale invariance linearly will also respect conformal symmetry [47], there exists no complete proof. The validity of this conjecture has been the subject of considerable interest in the recent literature [48], [49].

Consider a theory where conformal invariance is spontaneously broken. Then the low energy effective theory contains a dilaton field $\sigma(x)$, which can be thought of as the NGB associated with the breaking of scale invariance [22–25]. The additional four NGBs associated with the breaking of the special conformal symmetry can be identified with the derivatives of the dilaton, rather than as independent propagating fields. Below the breaking scale the symmetry is realized non-linearly, with the dilaton undergoing a shift $\sigma(x) \to \sigma'(x') = \sigma(x) + \omega f$ under the scale transformation $x^\mu \to x'^\mu = e^{-\omega} x^\mu$. Here f is the scale associated with the breaking of conformal symmetry. For the purpose of writing interactions of the dilaton it is convenient to define the object

$$\chi(x) = f e^{\sigma(x)/f} \tag{1}$$

which transforms linearly under scale transformations. Specifically, under the scale transformation $x^\mu \to x'^\mu = e^{-\omega} x^\mu$, $\chi(x)$ transforms as a conformal compensator

$$\chi(x) \to \chi'(x') = e^{\omega}\, \chi(x)\ . \tag{2}$$

The low energy effective theory for the dilaton will in general include all terms consistent with this transformation, but with some additional restrictions and relations among their coefficients from the requirement that the theory be invariant not just under scale transformations, but under the full conformal group. These restrictions will not affect our discussion in any significant way, and so operationally we shall only require that the action for χ be scale invariant.

In writing down the Lagrangian for the dilaton, it is necessary to take into account the implicit breaking of conformal invariance associated with the regulator. Since the theory in the ultraviolet possesses exact conformal invariance, this effect is of course completely spurious. However, it has the consequence that the Lagrangian for the dilaton is not manifestly scale invariant. It is

* A closely related result has been obtained in the context of walking gauge theories using current algebraic methods [30].

only at the quantum level, when effects of the regulator are incorporated, that conformal invariance is realized. This complicates the problem of finding the form of the effective theory.

Perhaps the simplest way to incorporate the effect of the regulator is to begin in a framework where the renormalization scale μ_χ is itself a function of χ, $\mu_\chi = \mu\hat{\chi}$, where $\hat{\chi} = \chi/f$. In such a framework, correlation functions can be obtained from the effective action, which has exactly the same form as in a conventional renormalization scheme, but with μ replaced by μ_χ [50]. Such a choice of renormalization scheme has the advantage that the action for χ is then manifestly scale invariant and therefore easy to write down. Starting from this action, the form of the Lagrangian in a more conventional scheme where the renormalization scale is independent of χ can be determined. This is the approach we shall follow.

A. Effective Theory in the Limit of Exact Conformal Invariance

We begin by constructing the effective theory for the dilaton in the case when conformal invariance is exact, and effects that explicitly violate conformal symmetry are absent. In a framework where the renormalization scale μ_χ is proportional to χ, the low energy effective action for the dilaton will be manifestly scale invariant. This symmetry allows derivative terms in the Lagrangian of the form

$$\frac{1}{2}Z\partial_\mu\chi\partial^\mu\chi + \frac{c}{\chi^4}\left(\partial_\mu\chi\partial^\mu\chi\right)^2 + \ldots \tag{3}$$

For reasons that will become clear, we postpone rescaling Z to one. Crucially, however, in contrast to the effective theory of the NGB of spontaneously broken global symmetry, a non-derivative term in the potential is also allowed,

$$V(\chi) = \frac{Z^2\kappa_0}{4!}\chi^4 \; . \tag{4}$$

The existence of this non-derivative term indicates that even in the absence of effects that explicitly violate conformal symmetry, there is a preferred value of $f = \langle\chi\rangle$. This is in sharp contrast to the case of a spontaneously broken global symmetry, where all points on the vacuum manifold are identical. In order to determine the location of the minimum, the effective potential must be obtained and minimized.

In order to bring the theory into a standard form, we now go over to a scheme in which the renormalization scale μ is independent of χ. In order to clarify the discussion, we first illustrate the procedure at one loop. We will obtain the Lagrangian for the low energy effective theory to this order, and use it to determine the effective potential and dilaton mass. We will then show how the result generalizes to arbitrary numbers of loops. It will be convenient to work in a mass-independent scheme, such as $\overline{\mathrm{MS}}$. We label Z and the coupling constants c, $Z^2\kappa_0$ etc. by g_i, where i is an index. The g_i are all dimensionless.

1. *One Loop Analysis*

At one loop, going over to a scheme where the renormalization scale μ is independent of χ is equivalent to evolving the parameters g_i etc. from μ_χ to μ using the renormalization group. Running the renormalization group leads to g_i evolving into g_i', where

$$g_i' = g_i - \frac{\mathrm{d}g_i}{\mathrm{d}\,\log\mu}\log\left(\frac{\chi}{f}\right) \; . \tag{5}$$

To keep the analysis simple we focus on the case when all the g_i are zero, except Z and $Z^2\kappa_0$. Then to this order, the potential for the dilaton takes the form

$$V(\chi) = \left\{Z^2\kappa_0 - \frac{\mathrm{d}(Z^2\kappa_0)}{\mathrm{d}\,\log\mu}\log\left(\frac{\chi}{f}\right)\right\}\frac{\chi^4}{4!} \; . \tag{6}$$

Note that the potential is no longer manifestly scale invariant. In this theory at one loop order there is no wave function renormalization,

$$\frac{\mathrm{d}\,\log Z}{\mathrm{d}\,\log\mu} = -2\gamma = 0 \; . \tag{7}$$

The derivative of κ_0 can be evaluated in perturbation theory, leading to

$$\frac{\mathrm{d}\kappa_0}{\mathrm{d}\,\log\mu} = \frac{3\kappa_0^2}{16\pi^2} \; . \tag{8}$$

After using these expressions to replace the terms involving derivatives in the Lagrangian, we may choose to rescale Z to one.

The conformal invariance of this Lagrangian can be made more transparent in a basis where all the mass scales are expressed as powers of the renormalization scale μ, and all coupling constants are dimensionless. In such a basis, the dilaton kinetic term can be written as

$$\frac{1}{2}\bar{Z}\partial_\mu\chi\partial^\mu\chi \; , \tag{9}$$

where $\bar{Z}$ is given to one loop order by

$$\bar{Z} = Z - \frac{\mathrm{d}Z}{\mathrm{d}\,\log\mu}\log\left(\frac{\mu}{f}\right) \; . \tag{10}$$

$\bar{Z}$, which is equal to Z since wave function renormalization vanishes to this order, is a renormalization group invariant and does not change with μ. At this point we choose to rescale $\bar{Z}$ to one.

In this basis the potential for the dilaton takes the form

$$V(\chi) = \left\{\overline{\kappa}_0 - \frac{\mathrm{d}(Z^2\kappa_0)}{\mathrm{d}\,\log\mu}\log\left(\frac{\chi}{\mu}\right)\right\}\frac{\chi^4}{4!} \; . \tag{11}$$

where $\overline{\kappa}_0$ is given by

$$\overline{\kappa}_0 = Z^2\kappa_0 - \frac{\mathrm{d}(Z^2\kappa_0)}{\mathrm{d}\ \log\mu}\log\left(\frac{\mu}{f}\right) \ . \tag{12}$$

Note that $\overline{\kappa}_0$, like $\bar{Z}$, is independent of the renormalization scale μ to this loop order, as dictated by conformal invariance.

The next step is to determine the one loop effective potential. This can be computed from Eq. (6), after rescaling Z to one, using the Coleman-Weinberg formula,

$$V_{\rm eff} = V \pm \frac{1}{64\pi^2}\sum_i M_i^4\left(\log\frac{M_i^2}{\mu^2} - \frac{1}{2}\right) \ . \tag{13}$$

Here the sum is over the field dependent masses of all the states in the theory, the sign being positive for bosons and negative for fermions. This leads to

$$V_{\rm eff}(\chi_{cl}) = \left\{\kappa_0 - \frac{3\kappa_0^2}{32\pi^2}\left[\log\left(\frac{\mu^2}{\frac{1}{2}\kappa_0 f^2}\right) - \frac{1}{2}\right]\right\}\frac{\chi_{cl}^4}{4!} \tag{14}$$

The conformal invariance of the theory can be made clear by rewriting this in terms of $\overline{\kappa}_0$. We obtain

$$V_{\rm eff}(\chi_{cl}) = \frac{\hat{\kappa}_0}{4!}\chi_{cl}^4 \ . \tag{15}$$

where $\hat{\kappa}_0$, given to one loop order by

$$\hat{\kappa}_0 = \overline{\kappa}_0 + \frac{3\overline{\kappa}_0^2}{32\pi^2}\left[\log\left(\frac{\overline{\kappa}_0}{2}\right) - \frac{1}{2}\right] \ , \tag{16}$$

is independent of the renormalization scale μ, as required by conformal invariance.

Minimizing this effective potential, we find that the conformal symmetry breaking scale $\langle\chi\rangle = f$ is driven to zero, corresponding to unbroken conformal symmetry, if the sign of $\hat{\kappa}_0$ is positive. Alternatively, if $\hat{\kappa}_0$ is negative, f is driven to infinite values, and conformal symmetry is never realized. Only if the value of $\hat{\kappa}_0$ is identically zero does the low energy effective theory possess a stable minimum, and a massless dilaton. In general setting $\hat{\kappa}_0 = 0$ is associated with tuning, since there is no symmetry reason to expect it to vanish.

2. *General Analysis*

Although this result was obtained based on a one loop analysis, we now show that the same conclusion holds at arbitrary loop order. It can be verified that by replacing g_i in the theory renormalized at μ_χ by g_i', where g_i' is given by

$$g_i' = g_i + \sum_{n=1}^{\infty}\frac{(-1)^n}{n!}\frac{\mathrm{d}^n g_i}{\mathrm{d}\ \log\mu^n}\left[\log\left(\frac{\chi}{f}\right)\right]^n \ , \tag{17}$$

we obtain a Lagrangian which is conformally invariant when renormalized at μ. The higher terms in this series are to be determined self-consistently order by order in perturbation theory. The potential for the dilaton now takes the form

$$V(\chi) = \left\{\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\frac{\mathrm{d}^n(Z^2\kappa_0)}{\mathrm{d}\ \log\mu^n}\left[\log\left(\frac{\chi}{f}\right)\right]^n\right\}\frac{\chi^4}{4!} \ . \tag{18}$$

As expected, the Lagrangian does not possess a manifestly scale invariant form. We can choose to rescale Z to one after the derivatives have been evaluated, but not before.

The conformal invariance of the theory can be made more transparent by going over to a basis where all the mass scales are expressed as powers of the renormalization scale μ, and all coupling constants are dimensionless. In such a basis, $\bar{Z}$, the coefficient of the dilaton kinetic term, is given by

$$\bar{Z} = \sum_{m=0}^{\infty}\frac{(-1)^m}{m!}\frac{\mathrm{d}^m Z}{\mathrm{d}\ \log\mu^m}\left[\log\left(\frac{\mu}{f}\right)\right]^m \ . \tag{19}$$

$\bar{Z}$ does not change with the renormalization scale μ, and we rescale it to one without loss of generality. The potential for the dilaton takes the form

$$V(\chi) = \left\{\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\overline{\kappa}_{0,n}\left[\log\left(\frac{\chi}{\mu}\right)\right]^n\right\}\frac{\chi^4}{4!} \ , \tag{20}$$

where $\overline{\kappa}_{0,n}$ is given by

$$\overline{\kappa}_{0,n} = \sum_{m=0}^{\infty}\frac{(-1)^m}{m!}\frac{\mathrm{d}^{m+n}(Z^2\kappa_0)}{\mathrm{d}\ \log\mu^{m+n}}\left[\log\left(\frac{\mu}{f}\right)\right]^m \ . \tag{21}$$

The beta functions of all the $\overline{\kappa}_{0,n}$ vanish by construction. This is a reflection of the conformal invariance of this theory. Going forward, we denote the $\overline{\kappa}_{0,n}$ and all the other coupling constants in this basis by $\bar{g}_i$, where i is an index. The beta functions of all the $\bar{g}_i$ vanish as a consequence of conformal invariance.

The next step is to obtain the effective potential $V_{\rm eff}(\chi_{cl})$ for this theory, and to minimize it. How is $V_{\rm eff}(\chi_{cl})$ to be determined? This time, rather than work directly from the Lagrangian, we employ the Callan-Symanzik equation for the effective potential,

$$\left\{\mu\frac{\partial}{\partial\mu} + \beta_i\frac{\partial}{\partial\bar{g}_i} - \gamma\chi_{cl}\frac{\partial}{\partial\chi_{cl}}\right\}V_{\rm eff}(\chi_{cl},\bar{g}_i,\mu) = 0 \ . \tag{22}$$

For a conformal theory, the beta functions $\beta_i(\bar{g}_i)$ vanish. The anomalous dimension γ of χ is also zero. Then the Callan-Symanzik equation reduces to

$$\mu\frac{\partial}{\partial\mu}V_{\rm eff}(\chi_{cl},\bar{g}_i,\mu) = 0 \ . \tag{23}$$

The effective potential is then constrained by dimensional analysis to be of the especially simple form

$$V_{\rm eff}(\chi_{cl}) = \frac{\hat{\kappa}_0}{4!}\chi_{cl}^4 \ , \tag{24}$$

where $\hat{\kappa}_0$ is a constant that depends on the $\bar{g}_i$, but is independent of μ. We see that the theory does not have a stable minimum unless $\hat{\kappa}_0 = 0$, when the potential vanishes identically. The results of our one loop analysis are therefore confirmed.

B. Incorporating Conformal Symmetry Violating Effects

The situation changes if effects that explicitly break conformal symmetry are present in the theory. Consider an operator $\mathcal{O}(x)$ of scaling dimension Δ added to the Lagrangian,

$$\mathcal{L} = \mathcal{L}_{\rm CFT} + \lambda_{\mathcal{O}}\mathcal{O}(x) \; . \tag{25}$$

Under $x \to x' = e^{-\omega}x$, the operator $\mathcal{O}(x) \to \mathcal{O}'(x') = e^{\omega\Delta}\mathcal{O}(x)$. It is convenient to define the dimensionless coupling constant $\hat{\lambda}_{\mathcal{O}} = \lambda_{\mathcal{O}}\mu^{\Delta-4}$. We choose to normalize the operator $\mathcal{O}(x)$ such that $\hat{\lambda}_{\mathcal{O}}$ of order one corresponds to conformal symmetry violation becoming strong, so that it can no longer be treated as a perturbation on the conformal dynamics. This implies that if $\hat{\lambda}_{\mathcal{O}} \ll 1$, it satisfies the renormalization group equation

$$\frac{\mathrm{d}\,\log\hat{\lambda}_{\mathcal{O}}}{\mathrm{d}\,\log\mu} = -(4-\Delta) \; . \tag{26}$$

We wish to determine the effect of this deformation on the form of the low energy effective theory. In order to do this, note that for small $\hat{\lambda}_{\mathcal{O}}$ the action remains formally invariant under $x \to x' = e^{-\omega}x$ provided $\lambda_{\mathcal{O}}$ is taken to be a spurion that transforms as

$$\lambda_{\mathcal{O}} \to \lambda'_{\mathcal{O}} = e^{(4-\Delta)\omega}\lambda_{\mathcal{O}}. \tag{27}$$

This implies that the effective theory for χ will also respect conformal symmetry if $\lambda_{\mathcal{O}}$ is treated as a spurion that transforms in this way.

In determining the low energy effective theory for the dilaton it is again simplest to begin in a framework where the renormalization scale depends on the conformal compensator as $\mu_\chi = \mu\hat{\chi}$, since the Lagrangian is then manifestly scale invariant. The potential for the dilaton is then

$$V(\chi) = \frac{Z^2\kappa_0}{4!}\chi^4 - \sum_{n=1}^{\infty}\frac{Z^{2-n\epsilon/2}\kappa_n}{4!}\lambda_{\mathcal{O}}^n\,\chi^{(4-n\epsilon)} \; , \tag{28}$$

where ϵ is defined as $4-\Delta$. The next step is to go over to a more conventional scheme where the renormalization scale μ is independent of χ.

1. *One Loop Analysis*

In order to clarify the discussion we will first work in the limit that $\hat{\lambda}_{\mathcal{O}} \ll 1$ at scales μ of order f, and determine the vacuum structure and the dilaton mass to one loop order. We will then relax the assumption on $\hat{\lambda}_{\mathcal{O}}$ and also generalize the result to an arbitrary number of loops.

Keeping only the leading order term in $\hat{\lambda}_{\mathcal{O}}$, the potential for the dilaton Eq. (28) simplifies to

$$V(\chi) = \frac{Z^2\kappa_0}{4!}\chi^4 - \frac{Z^{\Delta/2}\kappa_1}{4!}\lambda_{\mathcal{O}}\,\chi^{\Delta} \; , \tag{29}$$

where κ_0 and κ_1 are coupling constants. We can go over to a scheme where the renormalization scale is independent of χ by using the renormalization group. The potential then becomes, to one loop order,

$$\begin{aligned} V(\chi) \;=\; & \left\{Z^2\kappa_0 - \frac{\mathrm{d}(Z^2\kappa_0)}{\mathrm{d}\,\log\mu}\log\left(\frac{\chi}{f}\right)\right\}\frac{\chi^4}{4!} \\ & - \left\{Z^{\Delta/2}\kappa_1 - \frac{\mathrm{d}(Z^{\Delta/2}\kappa_1)}{\mathrm{d}\,\log\mu}\log\left(\frac{\chi}{f}\right)\right\}\frac{\lambda_{\mathcal{O}}\,\chi^{\Delta}}{4!} \; . \end{aligned} \tag{30}$$

To keep the analysis simple we focus on the case when all the g_i are zero, except Z, $Z^2\kappa_0$ and $Z^{\Delta/2}\kappa_1$. This theory does not experience wave function renormalization at one loop, and therefore the derivatives of Z in the expression above vanish. The derivatives of κ_0 and κ_1 can be evaluated in perturbation theory, leading to

$$\begin{aligned} \frac{\mathrm{d}\kappa_0}{\mathrm{d}\,\log\mu} &= \frac{3\kappa_0^2}{16\pi^2} \\ \frac{\mathrm{d}\kappa_1}{\mathrm{d}\,\log\mu} &= \frac{\Delta\,(\Delta-1)\,\kappa_1\kappa_0}{32\pi^2} \; . \end{aligned} \tag{31}$$

In order to understand how conformal symmetry is realized in this framework it is useful to go over to a basis where all mass scales are expressed in terms of the renormalization scale μ and all coupling constants are dimensionless. In this basis, $\bar{Z}$, the coefficient of the dilaton kinetic term is given by

$$\bar{Z} = Z - \frac{\mathrm{d}Z}{\mathrm{d}\,\log\mu}\log\left(\frac{\mu}{f}\right) \; . \tag{32}$$

$\bar{Z}$ is a renormalization group invariant. The absence of wave function renormalization in this theory at one loop means that $Z = \bar{Z}$ to this order. We choose to rescale $\bar{Z}$ to one.

The potential for the dilaton takes the form

$$\begin{aligned} V(\chi) \;=\; & \left\{\overline{\kappa}_0 - \frac{\mathrm{d}(Z^2\kappa_0)}{\mathrm{d}\,\log\mu}\log\left(\frac{\chi}{\mu}\right)\right\}\frac{\chi^4}{4!} \\ & - \left\{\overline{\kappa}_1 - \frac{\mathrm{d}(Z^{\Delta/2}\kappa_1)}{\mathrm{d}\,\log\mu}\log\left(\frac{\chi}{\mu}\right)\right\}\frac{\lambda_{\mathcal{O}}\chi^{\Delta}}{4!} \; . \end{aligned} \tag{33}$$

Here $\overline{\kappa}_0$ and $\overline{\kappa}_1$ are related to Z, κ_0 and κ_1 as

$$\begin{aligned} \overline{\kappa}_0 &= Z^2\kappa_0 - \frac{\mathrm{d}(Z^2\kappa_0)}{\mathrm{d}\,\log\mu}\log\left(\frac{\mu}{f}\right) \\ \overline{\kappa}_1 &= Z^{\Delta/2}\kappa_1 - \frac{\mathrm{d}(Z^{\Delta/2}\kappa_1)}{\mathrm{d}\,\log\mu}\log\left(\frac{\mu}{f}\right) \; . \end{aligned} \tag{34}$$

Note that $\lambda_{\mathcal{O}}$ is being treated as a spurion, not as a coupling constant, and therefore continues to carry mass dimension $4-\Delta$, equal to its spurious scaling dimension. The beta functions of $\overline{\kappa}_0$ and $\overline{\kappa}_1$ can be seen to vanish to one loop order by construction. This is a consequence of the (spurious) conformal symmetry.

The next step is to obtain the effective potential for the theory at one loop order. We again use Eq. (13), after rescaling Z to one, leading to

$$\begin{aligned} V_{\rm eff}(\chi_{cl}) \;=\; & \frac{\overline{\kappa}_0}{4!}\chi_{cl}^4 - \frac{\overline{\kappa}_1}{4!}\lambda_{\mathcal{O}}\chi_{cl}^{\Delta} + \\ & \frac{\overline{\kappa}_0}{4!}\left[\frac{3\overline{\kappa}_0}{32\pi^2}\chi_{cl}^4 - \frac{\lambda_{\mathcal{O}}\Delta\left(\Delta-1\right)\overline{\kappa}_1}{64\pi^2}\chi_{cl}^{\Delta}\right]\left[\Sigma - \frac{1}{2}\right] , \end{aligned} \tag{35}$$

where Σ is defined as

$$\Sigma = \log\left[\frac{\overline{\kappa}_0}{2} - \frac{\overline{\kappa}_1}{4!}\Delta\left(\Delta-1\right)\lambda_{\mathcal{O}}\chi_{cl}^{\Delta-4}\right] . \tag{36}$$

The next step is to find the minimum of this potential, and to obtain the dilaton mass. For simplicity, we neglect the loop suppressed terms on the second line of Eq. (35). We will later verify that including them does not alter our conclusions. The tree level potential admits a minimum when

$$f^{(\Delta-4)} = \frac{4\overline{\kappa}_0}{\overline{\kappa}_1\lambda_{\mathcal{O}}\Delta} . \tag{37}$$

The dilaton mass squared at the minimum, to this order, is given by

$$m_\sigma^2 = \frac{\overline{\kappa}_1}{4!}\lambda_{\mathcal{O}}\Delta(4-\Delta)f^{\Delta-2} = 4\frac{\overline{\kappa}_0}{4!}(4-\Delta)f^2 . \tag{38}$$

If the conformal field theory is weakly coupled, the parameters $\overline{\kappa}_0, \overline{\kappa}_1 \ll (4\pi)^2$, $\hat{\lambda}_{\mathcal{O}} \ll 1 \Rightarrow \lambda_{\mathcal{O}} f^{(\Delta-4)} \ll 1$, and the effective theory of the dilaton we have obtained is valid. Corrections to Eqs. (37) and (38) from the loop suppressed terms in Eq. (35) can be seen to be small in this limit, and we are justified in neglecting them.

However, if the conformal field theory under consideration is strongly coupled, as in the theories of interest for electroweak symmetry breaking, the effective theory of the dilaton is also expected to be strongly coupled at the scale $\Lambda \sim 4\pi f$. Then, in the absence of tuning, the parameters $\overline{\kappa}_0$ and $\overline{\kappa}_1$ are in general of order $(4\pi)^2$ and, as is clear from Eq. (37), the assumption that $\hat{\lambda}_{\mathcal{O}}$ is small at the scale f is no longer self consistent. Furthermore, it follows from Eq. (38) that the mass of the dilaton is of order the cutoff Λ and so it is no longer a light state. The loop suppressed terms we have neglected in obtaining Eq. (37) cannot alter this result. The conclusion to be drawn from this is that if a strongly coupled conformal field theory is explicitly broken by a relevant operator that becomes strong in the infrared, in general there is no reason to expect a light dilaton.

However, a closer study of Eq. (38) reveals a very interesting feature. If the operator $\mathcal{O}$ is very close to marginal so that $(4-\Delta) \ll 1$, then even for $\overline{\kappa}_0 \sim (4\pi)^2$ the dilaton mass is parametrically smaller than the strong coupling scale Λ. It is straightforward to verify that this conclusion remains true even when the loop suppressed terms in Eq. (35) are included in the analysis. This would suggest that in a scenario where the operator that breaks conformal symmetry is close to marginal, there is indeed a light dilaton in the effective theory. The dilaton mass depends on how close the dimension of $\mathcal{O}$ is to the exactly marginal value of 4, scaling as $m_\sigma \sim \sqrt{4-\Delta}$.

This is potentially a very important result. In a large class of theories of interest for electroweak symmetry breaking, the operator that breaks conformal symmetry is close to marginal in order to ensure that there is a large hierarchy between the flavor scale (or Planck scale) and the electroweak scale. This result would imply that in all such theories the low energy spectrum includes a light dilaton! Unfortunately, the steps leading up to Eq. (38) assumed that $\hat{\lambda}_{\mathcal{O}} \ll 1$. As is clear from Eq. (37), this assumption is not valid in the strong coupling limit. In order to validate this conclusion, we must show that the result continues to hold when this assumption is relaxed, and is valid beyond one loop.

2. *General Analysis*

Extending the analysis beyond small $\lambda_{\mathcal{O}}$ involves incorporating two distinct effects. Firstly, if the coupling constant $\lambda_{\mathcal{O}}$ is not small, the operator $\mathcal{O}(x)$ is expected to acquire an anomalous dimension and Eq. (26) is in general no longer valid. Instead, the renormalization group equation takes on the more general form

$$\frac{{\rm d}\,\log\hat{\lambda}_{\mathcal{O}}}{{\rm d}\,\log\mu} = -g(\hat{\lambda}_{\mathcal{O}}) , \tag{39}$$

where $g(\hat{\lambda}_{\mathcal{O}})$ is in general a polynomial in $\hat{\lambda}_{\mathcal{O}}$,

$$g(\hat{\lambda}_{\mathcal{O}}) = \sum_{n=0}^{\infty} c_n \hat{\lambda}_{\mathcal{O}}^n , \tag{40}$$

that can be approximated by the lowest order term

$$g(\hat{\lambda}_{\mathcal{O}}) = c_0 = (4-\Delta) \tag{41}$$

only in the limit when $\hat{\lambda}_{\mathcal{O}}$ is small. In general, in a strongly coupled conformal field theory, the coefficients c_n, $n \geq 1$ are expected to be of order one. (This is consistent with the expectation that all the terms in the series should become comparable when $\hat{\lambda}_{\mathcal{O}}$ is of order one.) This effect must be taken into account. Secondly, if $\hat{\lambda}_{\mathcal{O}}$ is not small, the higher order terms in Eq. (28) are significant and must be included in our analysis.

While both these effects are important, the first has a particularly striking impact on the form of the low energy effective theory. The reason is that in this case, the leading order effect which is of order $(4-\Delta)$ receives

corrections that begin at order $\hat{\lambda}_{\mathcal{O}}$. Since in the theories of interest $(4-\Delta) \ll 1$, these corrections can potentially become large even before $\hat{\lambda}_{\mathcal{O}}$ reaches its strong coupling value, significantly impacting the final result. This is most easily seen by going beyond the leading term in Eq. (40), while neglecting the higher order corrections in $\hat{\lambda}_{\mathcal{O}}$ that arise from other sources. Such an approximation is valid provided $\hat{\lambda}_{\mathcal{O}}$ at the breaking scale is significantly below its strong coupling value, $\hat{\lambda}_{\mathcal{O}} \ll 1$. We are interested in the region of parameter space where $(4-\Delta) < c_1\hat{\lambda}_{\mathcal{O}}$, so that the corrections to the leading order term in Eq. (40) dominate. We postpone a more complete discussion that includes all the higher order effects in $\hat{\lambda}_{\mathcal{O}}$ till later in this section.

Integrating Eq (39), it follows that $\mathcal{G}(\hat{\lambda}_{\mathcal{O}})\mu^{-1}$ is a renormalization group invariant, where

$$\mathcal{G}(\hat{\lambda}_{\mathcal{O}}) = \exp\left(-\int \frac{d\hat{\lambda}_{\mathcal{O}}}{\hat{\lambda}_{\mathcal{O}}}\frac{1}{g(\hat{\lambda}_{\mathcal{O}})}\right) . \tag{42}$$

We can make the theory defined by Eq. (25) formally invariant under scale transformations by promoting $\hat{\lambda}_{\mathcal{O}}$ to a spurion that transforms as

$$\hat{\lambda}_{\mathcal{O}}(\mu) \to \hat{\lambda}'_{\mathcal{O}}(\mu) = \hat{\lambda}_{\mathcal{O}}(\mu e^{-\omega}) \tag{43}$$

under $x \to x' = e^{-\omega}x$. Under this transformation,

$$\mathcal{G}(\hat{\lambda}_{\mathcal{O}})\mu^{-1} \to \mathcal{G}(\hat{\lambda}'_{\mathcal{O}})\mu^{-1} = e^{-\omega}\mathcal{G}(\hat{\lambda}_{\mathcal{O}})\mu^{-1} . \tag{44}$$

The Lagrangian for the low energy effective theory must be invariant under this spurious scale transformation. Furthermore, it is restricted to terms involving positive integer powers of the spurion $\hat{\lambda}_{\mathcal{O}}$. Using Eq. (42), it follows that the combination $\overline{\lambda}_{\mathcal{O}}$, defined as

$$\overline{\lambda}_{\mathcal{O}} = \hat{\lambda}_{\mathcal{O}}\left[1 + g(\hat{\lambda}_{\mathcal{O}})\log\mu\right] , \tag{45}$$

is invariant under infinitesimal changes in the renormalization scale μ. It then follows from Eq. (44) that the object $\overline{\Omega}(\hat{\lambda}_{\mathcal{O}}, \chi/\mu)$, defined as

$$\overline{\Omega}(\hat{\lambda}_{\mathcal{O}}, \chi/\mu) = \hat{\lambda}_{\mathcal{O}}\left[1 - g(\hat{\lambda}_{\mathcal{O}})\log\left(\frac{\chi}{\mu}\right)\right] , \tag{46}$$

is invariant under infinitesimal (spurious) scale transformations. Furthermore, $\overline{\Omega}$ is a polynomial in $\hat{\lambda}_{\mathcal{O}}$. Lagrangians that are invariant under infinitesimal (spurious) scale transformations can be constructed using $\overline{\Omega}$.

For values of μ close to the symmetry breaking scale f and $g(\hat{\lambda}_{\mathcal{O}}) \ll 1$, we can approximate $\overline{\Omega}$ as

$$\overline{\Omega}(\hat{\lambda}_{\mathcal{O}}, \chi/\mu) = \hat{\lambda}_{\mathcal{O}}\left(\frac{\chi}{\mu}\right)^{-g(\hat{\lambda}_{\mathcal{O}})} . \tag{47}$$

To leading order in $\overline{\Omega}$ the potential for χ takes the form

$$V(\chi) = \frac{\chi^4}{4!}\left(\kappa_0 - \kappa_1\overline{\Omega}\right) . \tag{48}$$

From this potential the dilaton mass at the minimum can be obtained as

$$m_\sigma^2 = 4\frac{\kappa_0}{4!}g(\hat{\lambda}_{\mathcal{O}})f^2 \tag{49}$$

This expression for the dilaton mass is very similar to that in Eq. (38), except in one important respect. We now see that it is the scaling dimension of the operator $\mathcal{O}$ at the breaking scale that determines the dilaton mass, rather than the scaling dimension of $\mathcal{O}$ in the far ultraviolet. In particular, this implies that for the dilaton of a spontaneously broken approximate conformal symmetry to be naturally light, it is not sufficient that $(4-\Delta) \ll 1$, so that the operator that breaks the symmetry is close to marginal in the far ultraviolet. Instead, the requirement is that this operator be close to marginal at the symmetry breaking scale, so that $g(\hat{\lambda}_{\mathcal{O}}) \ll 1$ at $\mu = f$. Since in a general strongly coupled theory, $\hat{\lambda}_{\mathcal{O}}$, and therefore $g(\hat{\lambda}_{\mathcal{O}})$, are expected to be of order one at the breaking scale, this condition is not expected to be satisfied in the scenarios of interest for electroweak symmetry breaking (for which $4-\Delta \ll 1$ suffices to address the flavor problem). This suggests that the existence of a light dilaton in these theories is associated with tuning (or more precisely, a coincidence problem). However, since the consistency of this analysis requires that $\hat{\lambda}_{\mathcal{O}} \ll 1$, it remains to show that including the higher order corrections in $\hat{\lambda}_{\mathcal{O}}$ that we have so far neglected, and which may be significant, does not affect this conclusion.

The next step is obtain the effective theory for the dilaton, consistently including all the higher order effects in $\hat{\lambda}_{\mathcal{O}}$. At this point it is convenient to separate out the contribution of the anomalous dimension of $\mathcal{O}$ to $g(\hat{\lambda}_{\mathcal{O}})$. Recalling that ϵ is defined as $(4-\Delta)$, we write

$$g(\hat{\lambda}_{\mathcal{O}}) = \epsilon + \delta g(\hat{\lambda}_{\mathcal{O}}) , \tag{50}$$

where $\delta g(\hat{\lambda}_{\mathcal{O}})$ is the anomalous dimension. In order to simplify our analysis we will consider the two cases $|\delta g(\hat{\lambda}_{\mathcal{O}})| < \epsilon$ and $|\delta g(\hat{\lambda}_{\mathcal{O}})| > \epsilon$, corresponding to the anomalous dimension of the operator $\mathcal{O}$ being smaller or larger than ϵ at the breaking scale, separately.

Limit When Anomalous Dimension of $\mathcal{O}$ Is Small

We first consider the case when $|\delta g(\hat{\lambda}_{\mathcal{O}})| < \epsilon$. In this limit we can simplify Eq. (42) by performing a binomial expansion,

$$\int d\hat{\lambda}_{\mathcal{O}}\frac{1}{\hat{\lambda}_{\mathcal{O}}g(\hat{\lambda}_{\mathcal{O}})} = \int\frac{d\hat{\lambda}_{\mathcal{O}}}{\epsilon\hat{\lambda}_{\mathcal{O}}}\left[1 - \frac{\delta g(\hat{\lambda}_{\mathcal{O}})}{\epsilon} + \ldots\right] . \tag{51}$$

Then

$$\mathcal{G}(\hat{\lambda}_{\mathcal{O}}) = \hat{\lambda}_{\mathcal{O}}^{-1/\epsilon}\exp\left[\int d\hat{\lambda}_{\mathcal{O}}\frac{\delta g(\hat{\lambda}_{\mathcal{O}})}{\epsilon^2\hat{\lambda}_{\mathcal{O}}} + \ldots\right] . \tag{52}$$

It follows from this that $\overline{\mathcal{G}}(\hat{\lambda}_{\mathcal{O}})$, defined as

$$\overline{\mathcal{G}}(\hat{\lambda}_{\mathcal{O}}) = \left[\mathcal{G}(\hat{\lambda}_{\mathcal{O}})\right]^{-\epsilon} , \tag{53}$$

can be expanded as a polynomial in $\hat{\lambda}_{\mathcal{O}}$,

$$\overline{\mathcal{G}}(\hat{\lambda}_{\mathcal{O}}) = \hat{\lambda}_{\mathcal{O}} \left[1 - \int d\hat{\lambda}_{\mathcal{O}} \frac{\delta g(\hat{\lambda}_{\mathcal{O}})}{\epsilon \hat{\lambda}_{\mathcal{O}}} + \ldots \right] . \tag{54}$$

Then the object $\overline{\mathcal{G}}(\hat{\lambda}_{\mathcal{O}})\mu^{\epsilon}$, which we denote by $\overline{\lambda}_{\mathcal{O}}$, is a renormalization group invariant that can be expanded as a polynomial in $\hat{\lambda}_{\mathcal{O}}$. It follows from Eq. (44) that the theory above the breaking scale is formally invariant under scale transformations, $x \to x' = e^{-\omega}x$, provided $\overline{\lambda}_{\mathcal{O}}$ is taken to be a spurion that transforms as

$$\overline{\lambda}_{\mathcal{O}} \to \overline{\lambda}'_{\mathcal{O}} = e^{\epsilon\omega}\overline{\lambda}_{\mathcal{O}}. \tag{55}$$

The effective theory for χ will then respect conformal symmetry if $\overline{\lambda}_{\mathcal{O}}$ is treated as a spurion that transforms in this way. Note that this spurious transformation is identical to that of $\lambda_{\mathcal{O}}$, Eq. (27), in the case of small $\hat{\lambda}_{\mathcal{O}}$.

Consider the object $\Omega(\overline{\lambda}_{\mathcal{O}}, \chi)$, defined as

$$\Omega(\overline{\lambda}_{\mathcal{O}}, \chi) = \overline{\lambda}_{\mathcal{O}} \chi^{-\epsilon} . \tag{56}$$

By construction, Ω is invariant under (spurious) scale transformations. Furthermore, in the regime $|\delta g(\hat{\lambda}_{\mathcal{O}})| < \epsilon$, it can be expanded as a polynomial in $\hat{\lambda}_{\mathcal{O}}$. Ω is useful in constructing the general Lagrangian for the low energy theory.

In a framework where the renormalization scale depends on the conformal compensator as $\mu_\chi = \mu\hat{\chi}$, the potential for χ takes the form

$$V(\chi) = \frac{Z^2\chi^4}{4!}\left[\kappa_0 - \sum_{n=1}^{\infty} \kappa_n \Omega^n(\overline{\lambda}_{\mathcal{O}}, \sqrt{Z}\chi)\right] . \tag{57}$$

This simplifies to the form of Eq (28), but with $\lambda_{\mathcal{O}}$ replaced by $\overline{\lambda}_{\mathcal{O}}$,

$$V(\chi) = \frac{Z^2\kappa_0}{4!}\chi^4 - \sum_{n=1}^{\infty} \frac{Z^{2-n\epsilon/2}\kappa_n}{4!} \overline{\lambda}^n_{\mathcal{O}}\, \chi^{(4-n\epsilon)} . \tag{58}$$

Going over to a more conventional scheme where the renormalization scale μ is independent of χ, $V(\chi)$ becomes

$$\sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \frac{\mathrm{d}^m (Z^2\kappa_0)}{\mathrm{d}\, \log\mu^m} \left[\log\left(\frac{\chi}{f}\right)\right]^m \frac{\chi^4}{4!} - \tag{59}$$
$$\sum_{n=1}^{\infty}\sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \frac{\mathrm{d}^m (Z^{2-n\epsilon/2}\kappa_n)}{\mathrm{d}\, \log\mu^m} \left[\log\left(\frac{\chi}{f}\right)\right]^m \frac{\overline{\lambda}^n_{\mathcal{O}}\chi^{4-n\epsilon}}{4!} .$$

We can choose to rescale Z to one, but only after the derivatives above have been evaluated.

The (spurious) conformal symmetry of the theory can be made more transparent in a basis where all mass scales are expressed in terms of the renormalization scale μ and all coupling constants are dimensionless. In this basis, $\bar{Z}$, the coefficient of the dilaton kinetic term, is given by

$$\bar{Z} = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \frac{\mathrm{d}^m Z}{\mathrm{d}\, \log\mu^m} \left[\log\left(\frac{\mu}{f}\right)\right]^m . \tag{60}$$

$\bar{Z}$ is independent of the renormalization scale μ. We again choose to set it to one. The potential for the dilaton now takes the form

$$V(\chi) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \overline{\kappa}_{0,m} \left[\log\left(\frac{\chi}{\mu}\right)\right]^m \frac{\chi^4}{4!} \tag{61}$$
$$- \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} \frac{(-1)^m}{m!} \overline{\kappa}_{n,m} \left[\log\left(\frac{\chi}{\mu}\right)\right]^m \frac{\overline{\lambda}^n_{\mathcal{O}}\chi^{4-n\epsilon}}{4!}$$

where the couplings constants $\overline{\kappa}_{n,m}$ are given by

$$\sum_{r=0}^{\infty} \frac{(-1)^r}{r!} \frac{\mathrm{d}^{m+r}(Z^{2-n\epsilon/2}\kappa_n)}{\mathrm{d}\, \log\mu^{m+r}} \left[\log\left(\frac{\mu}{f}\right)\right]^r . \tag{62}$$

The beta functions of all the $\overline{\kappa}_{n,m}$ vanish by construction, reflecting the (spurious) conformal invariance of the theory.

The final step is to determine the form of the effective potential. We will again use the Callan-Symanzik equation for the effective potential,

$$\left\{\mu\frac{\partial}{\partial\mu} + \beta_i \frac{\partial}{\partial \bar{g}_i} - \gamma_{\phi_\alpha}\phi_\alpha \frac{\partial}{\partial\phi_\alpha}\right\} V_{\mathrm{eff}}(\phi_\alpha, \bar{g}_i, \mu) = 0 . \tag{63}$$

Here the index α runs over the fields in the theory, namely χ_{cl} and $\hat{\lambda}_{\mathcal{O}}$. The beta functions $\beta_i(\bar{g}_i)$ vanish as a consequence of the (spurious) conformal symmetry, as does the anomalous dimension of χ. The anomalous dimension of $\hat{\lambda}_{\mathcal{O}}$ is $g(\hat{\lambda}_{\mathcal{O}})$, the difference between its scaling dimension and mass dimension. Then the Callan-Symanzik equation reduces to

$$\left\{\mu\frac{\partial}{\partial\mu} - g(\hat{\lambda}_{\mathcal{O}})\hat{\lambda}_{\mathcal{O}} \frac{\partial}{\partial\hat{\lambda}_{\mathcal{O}}}\right\} V_{\mathrm{eff}}(\chi_{cl}, \hat{\lambda}_{\mathcal{O}}, \bar{g}_i, \mu) = 0 . \tag{64}$$

Making a change of variable from $\hat{\lambda}_{\mathcal{O}}$ to $\overline{\lambda}_{\mathcal{O}}$, this becomes simply

$$\mu\frac{\partial}{\partial\mu} V_{\mathrm{eff}}(\chi_{cl}, \overline{\lambda}_{\mathcal{O}}, \bar{g}_i, \mu) = 0 . \tag{65}$$

Dimensional analysis constrains the solution to be of the form

$$V_{\mathrm{eff}}(\chi_{cl}) = \frac{1}{4!}\chi^4_{cl}\left[\hat{\kappa}_0 - \mathcal{F}(\overline{\lambda}_{\mathcal{O}}\chi_{cl}^{-\epsilon})\right] , \tag{66}$$

where $\hat{\kappa}_0$ is a constant that depends on the couplings $\bar{g}_i$ but not on $\overline{\lambda}_{\mathcal{O}}$. The form of the function $\mathcal{F}(\Omega)$ cannot be determined from symmetry considerations alone, but depends on the dynamics of the conformal field theory under consideration, and on the operator $\mathcal{O}$. For values of

Ω less than one by a factor of at least a few, corresponding to $\hat{\lambda}_{\mathcal{O}}$ being below its strong coupling value at the symmetry breaking scale f, $\mathcal{F}(\Omega)$ can be computed in perturbation theory. In general it is not a polynomial in Ω, as can be seen from Eq. (35).

Minimizing the effective potential we find the condition that determines the symmetry breaking scale f,

$$4\hat{\kappa}_0 - 4\mathcal{F}(\overline{\lambda}_{\mathcal{O}} f^{-\epsilon}) + \epsilon\overline{\lambda}_{\mathcal{O}} f^{-\epsilon}\mathcal{F}'(\overline{\lambda}_{\mathcal{O}} f^{-\epsilon}) = 0 \; . \tag{67}$$

The dilaton mass squared depends on the second derivative of the effective potential at the minimum, which is given by

$$\frac{\partial^2 V_{\mathrm{eff}}}{\partial\sigma^2} = \frac{1}{4!} f^2 \left\{ 4\epsilon\overline{\lambda}_{\mathcal{O}} f^{-\epsilon}\mathcal{F}'(\overline{\lambda}_{\mathcal{O}} f^{-\epsilon}) \right\} \; . \tag{68}$$

Here we are neglecting effects of order ϵ^2. We see from this that even at strong coupling, corresponding to $\hat{\kappa}_0 \sim (4\pi)^2$, if the function $\mathcal{F}(\Omega)$ satisfies the condition $\mathcal{F}(\Omega) \gtrsim \Omega\mathcal{F}'(\Omega)$ at the minimum, the dilaton mass is suppressed by a factor of $\sqrt{\epsilon}$ relative to the strong coupling scale $4\pi f$, and therefore remains light. The question is whether this condition on the function $\mathcal{F}(\Omega)$ is indeed satisfied in a general strongly coupled conformal field theory, for an arbitrary marginal operator $\mathcal{O}$. Unfortunately, in the absence of additional information about the function $\mathcal{F}(\Omega)$, we cannot establish such a conclusion. At the minimum, the value of Ω is equal to that of $\hat{\lambda}_{\mathcal{O}}$ evaluated at the symmetry breaking scale f. For Ω of order one, corresponding to $\hat{\lambda}_{\mathcal{O}}$ close to its strong coupling value, we expect that $\mathcal{F}(\Omega)$ is of order $(4\pi)^2$, but its functional form is completely unknown.

However, there exists a class of strongly coupled theories where the condition $\mathcal{F}(\Omega) \sim \Omega\mathcal{F}'(\Omega)$ is satisfied, and the dilaton is light. In the region of parameter space where $\hat{\lambda}_{\mathcal{O}}$ and $\hat{\kappa}_0$ are below their strong coupling values, the form of the function $\mathcal{F}(\Omega)$ can be determined from perturbation theory. In this regime it is dominated by the term linear in Ω in Eq. (57), since the other terms are loop suppressed or higher order in $\hat{\lambda}_{\mathcal{O}}$. Now, we expect that there exist strongly coupled conformal field theories where the parameter $\hat{\kappa}_0$ is below its natural strong coupling value by a factor of order a few. This is quite natural, requiring at most mild tuning. From the minimization condition it follows that in such theories, symmetry breaking is realized for values of $\mathcal{F}(\Omega)$ that correspond to values of Ω, and therefore $\hat{\lambda}_{\mathcal{O}}$, that lie below their strong coupling values by roughly the same factor. Since $\mathcal{F}(\Omega)$ is linear in Ω in this regime, the condition $\mathcal{F}(\Omega) \sim \Omega\mathcal{F}'(\Omega)$ is satisfied at the minimum. Therefore in this class of theories the conclusion $m_\sigma \sim \sqrt{\epsilon}$ is valid, and the dilaton is light.

Since this analysis is restricted to the region of parameter space where $|\delta g(\hat{\lambda}_{\mathcal{O}})| < \epsilon$, it is important to understand the circumstances under which this condition is satisfied. One possibility is that $\mathcal{O}$ is a protected operator, so that all the coefficients c_n in the polynomial expansion of $g(\hat{\lambda}_{\mathcal{O}})$ are of order ϵ. The operator $\mathcal{O}$ is then close to marginal for any value of $\hat{\lambda}_{\mathcal{O}}$. An example is a theory where the parameter $\hat{\lambda}_{\mathcal{O}}$ corresponds to a fixed line, while the parameter ϵ is associated with the coefficient of an operator that is very close to marginal (for all $\hat{\lambda}_{\mathcal{O}}$) and which lifts the fixed line. However, theories that admit such protected operators are clearly rather special. There is no reason to expect the condition $c_n \lesssim \epsilon$ to be satisfied by an arbitrary marginal operator $\mathcal{O}$ in a general conformal field theory.

Another possibility is that the parameter $\hat{\kappa}_0$ lies significantly below its natural strong coupling value so that symmetry breaking is realized for values of $\hat{\lambda}_{\mathcal{O}}$ less than ϵ. The condition $|\delta g(\hat{\lambda}_{\mathcal{O}})| < \epsilon$ can then be satisfied. Since in this regime $\mathcal{F}(\Omega)$ is dominated by the term linear in Ω, $\mathcal{F}(\Omega) \sim (4\pi)^2\Omega \sim (4\pi)^2\hat{\lambda}_{\mathcal{O}}$, it follows from Eq. (67) that the condition $\hat{\lambda}_{\mathcal{O}} < \epsilon$ translates into $\hat{\kappa}_0/(4\pi)^2 \lesssim \epsilon$. It follows from Eq. (68) and the minimization condition Eq. (67) that in this regime the dilaton mass scales as $\sqrt{\hat{\kappa}_0\epsilon}$, and therefore receives additional suppression from the fact that $\hat{\kappa}_0$ is small. However, small values of $\hat{\kappa}_0$ are associated with tuning, and so this condition is not expected to be satisfied in a general conformal field theory. However, in the case of small hierarchies, such as between the flavor scale and the weak scale, values of ϵ as large as 1/5 can still serve to address the problem. It follows from this that in such a theory, a dilaton mass a factor of 5 below the strong coupling scale can be realized for $\hat{\kappa}_0$ a factor of 5 below its natural strong coupling value. Since the tuning scales with $\hat{\kappa}_0$, this theory need only be tuned at the level of 1 part in 5 (20%). This is to be contrasted with the case of a (non-pNGB) composite scalar of the same mass, which is tuned at the level of 1 part in 25 (4%). We see that although this scenario is tuned, the tuning is mild, scaling with the mass of the dilaton rather than the square of its mass.

Limit When Anomalous Dimension of $\mathcal{O}$ Is Large

We now turn our attention to the case when the anomalous dimension of $\mathcal{O}$ is larger than ϵ in the neighbourhood of the breaking scale, so that $|\delta g(\hat{\lambda}_{\mathcal{O}})| > \epsilon$. For simplicity, we will work in the limit that the renormalization group evolution of $\log\hat{\lambda}_{\mathcal{O}}$ close to the breaking scale is dominated by the term linear in $\hat{\lambda}_{\mathcal{O}}$ so that

$$\frac{\mathrm{d}\,\log\hat{\lambda}_{\mathcal{O}}}{\mathrm{d}\,\log\mu} = -c_1\hat{\lambda}_{\mathcal{O}} \; , \tag{69}$$

Integrating this equation we find that $\mathcal{G}(\hat{\lambda}_{\mathcal{O}})$ is now given by

$$\mathcal{G}(\hat{\lambda}_{\mathcal{O}}) = \exp\left(\frac{1}{c_1\hat{\lambda}_{\mathcal{O}}}\right) \; . \tag{70}$$

Since $\mathcal{G}(\hat{\lambda}_{\mathcal{O}})\mu^{-1}$ is a renormalization group invariant, it

follows that $\overline{\lambda}_{\mathcal{O}}$, now defined as

$$\overline{\lambda}_{\mathcal{O}} = \frac{\hat{\lambda}_{\mathcal{O}}}{1 - c_1 \hat{\lambda}_{\mathcal{O}} \log\mu} \tag{71}$$

is also a renormalization group invariant. Once $\hat{\lambda}_{\mathcal{O}}$ is promoted to a spurion as in Eq (43), the object

$$\Omega(\overline{\lambda}_{\mathcal{O}}, \chi) = \frac{\overline{\lambda}_{\mathcal{O}}}{1 + c_1 \overline{\lambda}_{\mathcal{O}} \log\chi} \tag{72}$$

is invariant under (spurious) scale transformations. Furthermore, at scales μ close to $\langle\chi\rangle = f$, it can be expanded as a polynomial in $\hat{\lambda}_{\mathcal{O}}$. The Lagrangian for the low energy effective theory can be constructed using Ω.

In a scheme where the renormalization scale is proportional to χ, $\mu_\chi = \mu\hat{\chi}$, the potential takes the form

$$V(\chi) = \frac{Z^2\chi^4}{4!}\left[\kappa_0 - \sum_{n=1}^{\infty} \kappa_n \Omega^n(\overline{\lambda}_{\mathcal{O}}, \sqrt{Z}\chi)\right] . \tag{73}$$

It is straightforward to go over to a scheme where the renormalization scale μ is independent of χ, and where all mass parameters in the Lagrangian are expressed as powers of μ. As before, the dimensionless coupling constants $\bar{g}_i$ in such a scheme are independent of the renormalization scale μ.

The effective potential for the low energy effective theory can once again be determined from the Callan-Symanzik equation, Eq (63). The anomalous dimension of χ vanishes while that of the spurion $\hat{\lambda}_{\mathcal{O}}$ is given by $g(\hat{\lambda}_{\mathcal{O}}) = c_1\hat{\lambda}_{\mathcal{O}}$. As a consequence the Callan-Symanzik equation reduces to

$$\left\{\mu\frac{\partial}{\partial\mu} - c_1\hat{\lambda}_{\mathcal{O}}^2 \frac{\partial}{\partial\hat{\lambda}_{\mathcal{O}}}\right\} V_{\text{eff}}(\chi_{cl}, \hat{\lambda}_{\mathcal{O}}, \bar{g}_i, \mu) = 0 . \tag{74}$$

Making the change of variable from $\hat{\lambda}_{\mathcal{O}}$ to $\overline{\lambda}_{\mathcal{O}}$, this simplifies to

$$\mu\frac{\partial}{\partial\mu} V_{\text{eff}}(\chi_{cl}, \overline{\lambda}_{\mathcal{O}}, \bar{g}_i, \mu) = 0 . \tag{75}$$

Dimensional analysis constrains the solution to be of the form

$$V_{\text{eff}}(\chi_{cl}) = \frac{1}{4!}\chi_{cl}^4 \left\{\hat{\kappa}_0 - \mathcal{F}[\Omega(\overline{\lambda}_{\mathcal{O}}, \chi_{cl})]\right\} . \tag{76}$$

The form of the function $\mathcal{F}[\Omega]$ cannot be determined from symmetry considerations alone, but depends on the dynamics of the conformal field theory under consideration, and on the operator $\mathcal{O}$. For values of Ω less than one by at least a factor of a few, corresponding to $\hat{\lambda}_{\mathcal{O}}$ being below its strong coupling value at the symmetry breaking scale, $\mathcal{F}(\Omega)$ can be computed in perturbation theory.

Minimizing the effective potential we obtain the condition that determines f,

$$4\hat{\kappa}_0 - 4\mathcal{F}[\Omega] + c_1\Omega^2\mathcal{F}'[\Omega] = 0 . \tag{77}$$

The dilaton mass squared depends on the second derivative of the effective potential at the minimum, which can be determined as

$$\frac{\partial^2 V_{\text{eff}}}{\partial\sigma^2} = \frac{c_1}{4!} f^2\Omega^2 \left[(4 - 2c_1\Omega)\,\mathcal{F}' - c_1\Omega^2\mathcal{F}''\right] . \tag{78}$$

Once again we focus on theories where the parameter $\hat{\kappa}_0$ is below its natural strong coupling value by some factor, which could be as small as a few. In such theories, symmetry breaking is realized for values of $\hat{\lambda}_{\mathcal{O}}$ that are below its strong coupling value. In this limit the form of the effective potential can be determined in perturbation theory, and $\mathcal{F}(\Omega)$ is dominated by the term linear in Ω. Then at the minimum the condition $\mathcal{F}[\Omega] \sim \Omega\mathcal{F}'[\Omega]$ is satisfied. Noting that at the minimum the value of Ω is equal to that of $\hat{\lambda}_{\mathcal{O}}$ at the scale f, it follows from Eq. (77) and Eq. (78) that the dilaton mass squared scales as $m_\sigma^2 \sim c_1\hat{\lambda}_{\mathcal{O}}\hat{\kappa}_0$.

We see from this that the dilaton mass depends on the anomalous dimension $c_1\hat{\lambda}_{\mathcal{O}}$ of the operator $\mathcal{O}$ at the symmetry breaking scale. Since the minimization condition Eq. (77) relates Ω (and therefore $\hat{\lambda}_{\mathcal{O}}$ at the breaking scale) to $\hat{\kappa}_0$, for c_1 of order its natural value of one we have that the dilaton mass squared scales as $\hat{\kappa}_0^2$. This suggests that for $\hat{\kappa}_0$ of order its natural strong coupling value the dilaton mass lies near the cutoff of the theory, and it is not a light state. It follows from this that in general, the spectrum of a conformal field theory broken by an arbitrary marginal operator that grows strong in the infrared does not include a light dilaton.

The low energy effective theory will however contain a light dilaton if the parameter $\hat{\kappa}_0$ lies significantly below its natural strong coupling value. In general, this involves tuning, since this condition is not expected to be satisfied in an arbitrary conformal field theory. However, the tuning is mild, scaling with $\hat{\kappa}_0$ and therefore as the mass of the dilaton, so that a dilaton that lies a factor of 5 below the strong coupling scale is only tuned at the level of 1 part in 5 (20 %).

It follows from this discussion that in strongly coupled theories where an approximate conformal symmetry is spontaneously broken, the low energy spectrum includes a light dilaton if the operator that breaks the symmetry is close to marginal at the breaking scale. This condition is in general not expected to be satisfied by the theories of interest for electroweak symmetry breaking, and so the presence of a light dilaton in these theories is associated with tuning. However, the tuning is mild, scaling as the mass of the dilaton rather than as the square of its mass.

III. DILATON INTERACTIONS IN A CONFORMAL SM

In the limit that conformal invariance is exact, the form of the dilaton interactions with SM fields in the low energy effective theory is fixed by the requirement that

the symmetry be realized nonlinearly. However, in the scenario of interest, we expect significant deviations from exact conformal invariance because effects associated with the operator $\mathcal{O}$ that violate the symmetry are large at the breaking scale. It is crucial to understand the size of these effects, and the extent to which the predictions of the theory with exact conformal invariance are affected.

In this section, we consider a scenario where the SM gauge bosons and matter fields are all composites of a strongly interacting conformal sector that breaks electroweak symmetry dynamically, and there is no light Higgs. The AdS/CFT correspondence relates this scenario to Higgsless Randall-Sundrum models where the SM matter and gauge fields are localized on the infrared brane. The couplings of the dilaton to the SM fields in such a framework have been determined [27], and agree with earlier results for the couplings of the radion to brane-localized fields in Randall-Sundrum models [42–44]. Several authors have studied the question of distinguishing the dilaton from the Higgs at the LHC in such a scenario [51–53], see also [54]. We will study the corrections to the dilaton couplings in this scenario when effects associated with the operator $\mathcal{O}$ that explicitly violates conformal symmetry are incorporated.

We begin by considering the dilaton couplings to the W and Z gauge bosons. We choose to work in a basis where we write all gauge kinetic terms in the form

$$-\frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu}\ . \tag{79}$$

In the absence of conformal symmetry violating effects, the couplings of the dilaton to the W are such as to compensate for the breaking of conformal invariance by the gauge boson mass term. In unitary gauge these take the form

$$\left(\frac{\chi}{f}\right)^2\frac{m_W^2}{g^2}W^+_\mu W^{\mu-} \tag{80}$$

in the Lagrangian. Here m_W is the W gauge boson mass. Expanding the compensator $\chi = fe^{\sigma/f}$ out in terms of σ to leading order in inverse powers of f, we find for the dilaton couplings

$$2\frac{\sigma}{f}\frac{m_W^2}{g^2}W^+_\mu W^{\mu-}\ . \tag{81}$$

Next we consider the corrections to the dilaton couplings when conformal symmetry violating effects are included. We will focus on the case when the anomalous dimension of $\mathcal{O}$, $|\delta g(\hat{\lambda}_{\mathcal{O}})|$, is less than ϵ at the breaking scale. We will later argue that the same conclusions are obtained in the limit when $|\delta g(\hat{\lambda}_{\mathcal{O}})|$ is greater than ϵ.

The presence of conformal symmetry violating effects allows additional two derivative terms in the dilaton action,

$$\frac{1}{2}\left[1+\sum_{n=1}^{\infty}\alpha_{\chi,n}\overline{\lambda}^n_{\mathcal{O}}\ \chi^{(-n\epsilon)}\right]\partial_\mu\chi\partial^\mu\chi\ . \tag{82}$$

The dimensionless parameters $\alpha_{\chi,n}$ depend both on the operator $\mathcal{O}$ and the specific conformal field theory under consideration. They are expected to be of order one. These new terms contribute to the dilaton kinetic term, which now becomes

$$\frac{1}{2}\left[1+\sum_{n=1}^{\infty}\alpha_{\chi,n}\overline{\lambda}^n_{\mathcal{O}}\ f^{(-n\epsilon)}\right]\partial_\mu\sigma\partial^\mu\sigma\ . \tag{83}$$

When σ is rescaled to make the dilaton kinetic term canonical, we see that the effective impact of these terms is to alter the effective value of f in Eq. (81), while leaving the form of the interaction unchanged. More generally, it follows that corrections to the dilaton kinetic term from conformal symmetry violating effects do not alter the form of the dilaton couplings to the SM fields. Instead, to leading order in σ/f, they lead to a universal rescaling in the effective value of f, leaving the relative strengths of the dilaton couplings to the various SM fields unchanged. Since to the order we are working this effect can be entirely absorbed into the parameter f, we will not consider it further.

The gauge kinetic term also receives corrections from conformal symmetry violating effects. It now takes the form

$$-\frac{1}{4\hat{g}^2}\left[1+\sum_{n=1}^{\infty}\alpha_{W,n}\overline{\lambda}^n_{\mathcal{O}}\ \chi^{(-n\epsilon)}\right]F_{\mu\nu}F^{\mu\nu}\ , \tag{84}$$

where the parameters $\alpha_{W,n}$ are dimensionless. They are expected to be of order one. The physical gauge coupling is now given by

$$\frac{1}{g^2}=\frac{1}{\hat{g}^2}\left[1+\sum_{n=1}^{\infty}\alpha_{W,n}\overline{\lambda}^n_{\mathcal{O}}\ f^{(-n\epsilon)}\right]\ . \tag{85}$$

Expanding Eq. (84) to leading order in σ we obtain

$$\epsilon\frac{\bar{c}_W}{4g^2}\frac{\sigma}{f}F_{\mu\nu}F^{\mu\nu}\ , \tag{86}$$

where the dimensionless parameter $\bar{c}_W$ is given by

$$\bar{c}_W=\frac{\sum_{n=1}^{\infty}n\alpha_{W,n}\overline{\lambda}^n_{\mathcal{O}}\ f^{(-n\epsilon)}}{1+\sum_{n=1}^{\infty}\alpha_{W,n}\overline{\lambda}^n_{\mathcal{O}}\ f^{(-n\epsilon)}}\ . \tag{87}$$

In a strongly coupled theory $\bar{c}_W$ is expected to be of order $\overline{\lambda}_{\mathcal{O}}f^{-\epsilon}$, which is the value of $\hat{\lambda}_{\mathcal{O}}$ at the breaking scale f. It follows that this correction to the dilaton coupling is suppressed by $\epsilon\hat{\lambda}_{\mathcal{O}}$, which is of order m_σ^2/Λ^2.

Conformal symmetry violating effects also modify the gauge boson mass term, Eq. (80), which now becomes

$$\left(\frac{\chi}{f}\right)^2\left[1+\sum_{n=1}^{\infty}\beta_{W,n}\overline{\lambda}^n_{\mathcal{O}}\ \chi^{(-n\epsilon)}\right]\frac{\hat{m}_W^2}{\hat{g}^2}W^+_\mu W^{\mu-}\ . \tag{88}$$

Here $\hat{m}_W$ is the W boson mass in the unperturbed theory, and the dimensionless parameters $\beta_{W,n}$ are of order one.

Expanding this out in terms of $\sigma(x)$, we see that to leading order in inverse powers of f, the dilaton couples as

$$\frac{\sigma}{f}\frac{m_W^2}{g^2}\left[2+c_W\epsilon\right]W_\mu^+W^{\mu-}\ . \tag{89}$$

Here m_W^2 is again the physical W boson mass,

$$m_W^2=\hat{m}_W^2\left[\frac{1+\sum_{n=1}^\infty\beta_{W,n}\overline{\lambda}_{\mathcal{O}}^n\ f^{(-n\epsilon)}}{1+\sum_{n=1}^\infty\alpha_{W,n}\overline{\lambda}_{\mathcal{O}}^n\ f^{(-n\epsilon)}}\right]\ , \tag{90}$$

while the dimensionless parameter c_W is given by

$$c_W=-\frac{\sum_{n=1}^\infty n\beta_{W,n}\overline{\lambda}_{\mathcal{O}}^n\ f^{(-n\epsilon)}}{1+\sum_{n=1}^\infty\beta_{W,n}\overline{\lambda}_{\mathcal{O}}^n\ f^{(-n\epsilon)}}\ . \tag{91}$$

In the strong coupling limit, c_W is expected to be of order $\overline{\lambda}_{\mathcal{O}}f^{-\epsilon}\sim\hat{\lambda}_{\mathcal{O}}$. We see that the effect of the conformal symmetry violating term is to correct the dilaton couplings by order $\epsilon\hat{\lambda}_{\mathcal{O}}\sim m_\sigma^2/\Lambda^2$.

If we instead consider the limit when $|\delta g(\hat{\lambda}_{\mathcal{O}})|$ is greater than ϵ at the symmetry breaking scale f, the analysis is very similar. The only significant difference is that $\overline{\lambda}_{\mathcal{O}}\chi^{-\epsilon}$ in Eqs. (82), (84) and (88) is replaced by $\Omega(\overline{\lambda}_{\mathcal{O}},\chi)$, which in this limit is given by

$$\Omega(\overline{\lambda}_{\mathcal{O}},\chi)=\frac{\overline{\lambda}_{\mathcal{O}}}{1+c_1\overline{\lambda}_{\mathcal{O}}\log\chi}\ . \tag{92}$$

Following exactly the same sequence of steps we find that the corrections to the dilaton couplings have the same form, but are now suppressed by $c_1\Omega^2(\overline{\lambda}_{\mathcal{O}},f)$ rather than $\epsilon\overline{\lambda}_{\mathcal{O}}f^{-\epsilon}$. However, this new suppression factor is of order m_σ^2/Λ^2, exactly as before. We see that the corrections have the same form and are of the same size as in the case $|\delta g(\hat{\lambda}_{\mathcal{O}})|<\epsilon$. It is not difficult to verify that this result is quite general. Therefore, in the remainder of the paper we will limit our analysis to the case $|\delta g(\hat{\lambda}_{\mathcal{O}})|<\epsilon$, with the understanding that the same general conclusions apply to the case $|\delta g(\hat{\lambda}_{\mathcal{O}})|>\epsilon$ as well.

Next we turn our attention to the dilaton couplings to the massless gauge bosons of the SM, the photon and the gluon. Unlike the W and Z, the Lagrangian for these particles does not break conformal invariance at the classical level, only at the quantum level. At one loop the renormalization group equations for the corresponding gauge couplings are of the form

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu}\frac{1}{g^2}=\frac{b_<}{8\pi^2} \tag{93}$$

where the constant $b_<=-11/3$ for electromagnetism and $+7$ for color, at scales above the mass of the top. This implies that under infinitesimal scale transformations $x\to x'=e^{-\omega}x$, the operator $F_{\mu\nu}F^{\mu\nu}$ transforms as $F_{\mu\nu}F^{\mu\nu}(x)\to F'_{\mu\nu}F'^{\mu\nu}(x')$, where

$$F'_{\mu\nu}F'^{\mu\nu}(x')=e^{4\omega}\left(1+\frac{b_<}{8\pi^2}g^2\omega\right)F_{\mu\nu}F^{\mu\nu}(x) \tag{94}$$

If conformal symmetry is to be realized nonlinearly, the couplings of the dilaton must be such as to compensate for this. It is then easy to see that the dilaton couplings in the Lagrangian must take the form

$$\frac{b_<}{32\pi^2}\log\left(\frac{\chi}{f}\right)F_{\mu\nu}F^{\mu\nu}\ . \tag{95}$$

Expanding this out in terms of $\sigma(x)$, to leading order in inverse powers of f, we find for the dilaton coupling

$$\frac{b_<}{32\pi^2}\frac{\sigma}{f}F_{\mu\nu}F^{\mu\nu}\ . \tag{96}$$

It follows from this that the dilaton couples much more weakly to the massless gauge bosons than to the W or the Z. The reason is that the gauge interactions correspond to marginal operators in the low energy effective theory, while mass terms for the gauge bosons are relevant operators. Since the dilaton couples as a conformal compensator, it is to be expected that its couplings to massless gauge bosons are suppressed.

We now consider corrections to this interaction arising from conformal symmetry violating effects. These allow direct couplings of the compensator χ to the gauge kinetic term of the form

$$-\frac{1}{4\hat{g}^2}\left[1+\sum_{n=1}^\infty\alpha_{A,n}\overline{\lambda}_{\mathcal{O}}^n\ \chi^{(-n\epsilon)}\right]F_{\mu\nu}F^{\mu\nu}\ . \tag{97}$$

The physical gauge coupling is now given by

$$\frac{1}{g^2}=\frac{1}{\hat{g}^2}\left[1+\sum_{n=1}^\infty\alpha_{A,n}\overline{\lambda}_{\mathcal{O}}^n\ f^{(-n\epsilon)}\right]\ . \tag{98}$$

Expanding Eq. (97) in terms of σ, and combining with Eq. (96) we find for the dilaton coupling to massless gauge bosons

$$\frac{\sigma}{f}\left[\frac{b_<}{32\pi^2}+\frac{c_A}{4g^2}\epsilon\right]F_{\mu\nu}F^{\mu\nu}\ . \tag{99}$$

Here the dimensionless coupling c_A is given by

$$c_A=\frac{\sum_{n=1}^\infty n\alpha_{A,n}\overline{\lambda}_{\mathcal{O}}^n\ f^{(-n\epsilon)}}{1+\sum_{n=1}^\infty\alpha_{A,n}\overline{\lambda}_{\mathcal{O}}^n\ f^{(-n\epsilon)}}\ . \tag{100}$$

In a strongly coupled theory the parameter c_A is expected to be of order $\overline{\lambda}_{\mathcal{O}}f^{-\epsilon}\sim\hat{\lambda}_{\mathcal{O}}$ at the scale f. We see from this that the corrections to the dilaton coupling arising from symmetry breaking effects are suppressed by m_σ^2/Λ^2. Nevertheless, the fact that the leading order effect is loop suppressed and therefore small implies that the symmetry breaking contribution may dominate.

Finally we consider the couplings of the dilaton to the SM fermions. In the limit that conformal symmetry is exact, the coupling of the dilaton is such as to compensate for the spontaneous breaking of conformal

invariance by the fermion mass terms. These interactions take the form

$$\frac{\chi}{f} m_\psi \bar{\psi}\psi \tag{101}$$

in the potential, where we have suppressed flavor indices. Expanding the compensator out in terms of σ we obtain

$$\sigma \frac{m_\psi}{f} \bar{\psi}\psi \ . \tag{102}$$

However, if a conformal symmetry breaking effect of the form considered in the previous section is present, Eq. (101) generalizes to

$$\frac{\chi}{f}\left[1 + \sum_{n=1}^{\infty} \beta_{\psi,n} \overline{\lambda}^{n}_{\mathcal{O}} \, \chi^{(-n\epsilon)}\right] \hat{m}_\psi \bar{\psi}\psi \ , \tag{103}$$

where $\hat{m}_\psi$ is the fermion mass in the unperturbed theory, and the parameters $\beta_{\psi,n}$ are dimensionless. In obtaining this we have assumed that the operator $\mathcal{O}$ does not violate the approximate $\mathrm{U(3)}^5$ flavor symmetry associated with the SM fermions in the chiral limit, which is broken by the fermion mass terms. This ensures that the dilaton couples diagonally in the mass basis.

In general the operator $\mathcal{O}$ will also correct the fermion kinetic term, which generalizes to

$$\left[1 + \sum_{n=1}^{\infty} \alpha_{\psi,n} \overline{\lambda}^{n}_{\mathcal{O}} \, \chi^{(-n\epsilon)}\right] \overline{\psi}\gamma^\mu \partial_\mu \psi \ . \tag{104}$$

After expanding out Eqs. (103) and (104) in terms of σ, rescaling to make the fermion kinetic term canonical, and then using the equation of motion for ψ, we obtain a correction to the dilaton coupling of the form

$$\sigma \frac{m_\psi}{f} \left[1 + c_\psi \epsilon\right] \bar{\psi}\psi \ , \tag{105}$$

In a strongly coupled conformal field theory we expect that c_ψ is of order $\overline{\lambda}_{\mathcal{O}} f^{-\epsilon}$. We conclude from this that the effect of the conformal symmetry violating terms is to modify the dilaton couplings to the SM fermions by order m_σ^2/Λ^2.

In summary we see that corrections to the form of the dilaton couplings to SM states arising from conformal symmetry violating effects are suppressed by the square of the ratio of the dilaton mass to the strong coupling scale, and therefore under good theoretical control in the theories of interest. These contributions are generally subleading, except in the case of dilaton couplings to marginal operators, when symmetry violating effects can dominate.

IV. TECHNICOLOR

In this section we determine the form of the couplings of a light dilaton to the SM fields in a scenario where electroweak symmetry is broken dynamically by a strongly interacting sector, and there is no light Higgs. The strongly interacting sector is assumed to be conformal in the far ultraviolet. However, conformal symmetry is explicitly broken by the operator $\mathcal{O}$, which grows large close to the TeV scale triggering electroweak symmetry breaking. The SM gauge fields do not constitute part of the strongly interacting sector. However, this sector transforms under the weak and electromagnetic gauge interactions. It may also transform under the SM color group. The SM gauge interactions constitute another small explicit breaking of the conformal symmetry. The SM fermions may be elementary, or may emerge as composites or partial composites of the strong dynamics.

The AdS/CFT correspondence relates this class of theories to Higgsless Randall-Sundrum models with the SM gauge fields propagating in the bulk. The couplings of the radion to SM fields in this framework have been determined, in the limit that effects associated with the dynamics that stabilizes the radion are neglected [45, 46]. We reproduce these results, and in doing so establish their validity beyond the large N limit. We also determine the corrections to the dilaton couplings that arise from conformal symmetry violating effects.

In order to avoid large corrections to precision electroweak observables, the strongly interacting sector must respect a custodial SU(2) symmetry. This symmetry is not exact, but is broken by the SM Yukawa couplings, and also by hypercharge. A simple way to realize custodial symmetry is to extend the $\mathrm{SU(2)_L}$ symmetry of the SM to $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R}$. Only the diagonal generator of this new $\mathrm{SU(2)_R}$ is gauged, and is associated with hypercharge. The strong dynamics breaks this extended symmetry down to the diagonal SU(2), which is identified with the custodial symmetry. Only a U(1) subgroup of the original SU(2)$\times$ U(1) gauge symmetry survives, and is identified with electromagnetism.

The NGBs $\pi(x)$ that arise from the breaking of SU(2)$\times$ SU(2) gauge symmetry down to the custodial SU(2) can be parametrized in terms of a matrix Σ, defined as

$$\Sigma = e^{i\frac{\pi(x)}{v}} \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix} \ . \tag{106}$$

Here v is the electroweak VEV. Σ transforms linearly under $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R}$, and is therefore more convenient for writing interactions. In unitary gauge the NGBs $\pi(x)$ are absorbed into the W and Z gauge bosons and Σ can be replaced by its VEV.

A. Couplings to Gauge Bosons

We begin by determining the dilaton couplings to the W and Z gauge bosons. In the ultraviolet, the SM gauge interactions do not violate the conformal symmetry of the theory at the classical level, only at the quantum level. Therefore, when these effects are included, the theory still respects conformal symmetry up to effects which are suppressed by loops involving the SM gauge bosons.

Therefore, the dominant interactions of the dilaton to the W and Z bosons in the low energy effective theory arise from couplings which compensate for the breaking of conformal invariance by the gauge boson mass terms. These take exactly the same form in the Lagrangian as in the case of composite W and Z gauge bosons

$$\left(\frac{\chi}{f}\right)^2 \frac{m_W^2}{g^2} W_\mu^+ W^{\mu -} \ . \tag{107}$$

Expanding this out in terms of σ to leading order in inverse powers of f, we again find

$$2\frac{\sigma}{f}\frac{m_W^2}{g^2} W_\mu^+ W^{\mu -} \ . \tag{108}$$

This agrees with the known results for the coupling of the radion to bulk gauge bosons in Randall-Sundrum models [45, 46]. Our analysis shows that this formula is valid beyond the large N limit.

When conformal symmetry violating effects associated with the operator $\mathcal{O}$ are present, the gauge boson mass will in general depend on $\hat{\lambda}_{\mathcal{O}}$. Then Eq. 107) generalizes to

$$\left(\frac{\chi}{f}\right)^2 \left[1 + \sum_{n=1}^{\infty} \beta_{W,n} \overline{\lambda}_{\mathcal{O}}^n \, \chi^{(-n\epsilon)}\right] \frac{\hat{m}_W^2}{\hat{g}^2} W_\mu^+ W^{\mu -} \ . \tag{109}$$

Here $\hat{m}_W$ is the W boson mass in the unperturbed theory, and the dimensionless parameters $\beta_{W,n}$ are of order one. Expanding this out in terms of $\sigma(x)$, we find that the dilaton couples as

$$\frac{\sigma}{f}\frac{m_W^2}{g^2}\left[2 + c_W \epsilon\right] W_\mu^+ W^{\mu -} \ , \tag{110}$$

where m_W^2 is the physical W boson mass. The dimensionless parameter c_W is of order $\overline{\lambda}_{\mathcal{O}} f^{-\epsilon} \sim \hat{\lambda}_{\mathcal{O}}$ at the scale f, so that the correction to the coupling is suppressed by m_σ^2/Λ^2.

We move on to consider the dilaton couplings to the massless gauge bosons of the SM, the gluon and the photon. We first determine the form of the couplings in the limit that effects arising from the operator $\mathcal{O}$ are neglected. Above the breaking scale, the renormalization group equation for the corresponding gauge coupling takes the form

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu}\frac{1}{g_{UV}^2} = \frac{b_>}{8\pi^2} \ , \tag{111}$$

where the constant $b_>$ receives contributions from both elementary states and the strongly interacting sector. Similarly, below the breaking scale it takes the form

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu}\frac{1}{g_{IR}^2} = \frac{b_<}{8\pi^2} \tag{112}$$

where $b_<$ receives contributions from elementary states, and also from any additional light states that emerge from the strongly interacting sector after symmetry breaking.

Equation (111) indicates that above the symmetry breaking scale, under infinitesimal scale transformations $x \to x' = e^{-\omega}x$, the operator corresponding to the gauge kinetic term transforms as $F_{\mu\nu}F^{\mu\nu}(x) \to F'_{\mu\nu}F'^{\mu\nu}(x')$, where

$$F'_{\mu\nu}F'^{\mu\nu}(x') = e^{4\omega}\left(1 + \frac{b_>}{8\pi^2}g_{UV}^2\omega\right)F_{\mu\nu}F^{\mu\nu}(x) \tag{113}$$

Below the symmetry breaking scale the corresponding transformation is $F_{\mu\nu}F^{\mu\nu}(x) \to F'_{\mu\nu}F'^{\mu\nu}(x')$, where

$$F'_{\mu\nu}F'^{\mu\nu}(x') = e^{4\omega}\left(1 + \frac{b_<}{8\pi^2}g_{IR}^2\omega\right)F_{\mu\nu}F^{\mu\nu}(x) \tag{114}$$

Above the symmetry breaking scale we can make the gauge kinetic term formally invariant under infinitesimal scale transformations by promoting the gauge coupling constant g_{UV} to a spurion that under $x \to x' = e^{-\omega}x$ transforms as

$$\frac{1}{g_{UV}^2} \to \frac{1}{g'^2_{UV}} = \frac{1}{g_{UV}^2} - \frac{b_>}{8\pi^2}\omega \ . \tag{115}$$

Now, matching at one loop across the symmetry breaking threshold we have

$$\frac{1}{g_{IR}^2} = \frac{1}{g_{UV}^2} + \frac{C}{8\pi^2} \tag{116}$$

where C is a dimensionless number that depends on the gauge quantum numbers of the states in the strongly interacting sector that have been integrated out at the threshold. While C cannot be calculated, since it depends on details of the strong dynamics, it is of order the number of states that have masses at the threshold. It is independent of g^2 up to corrections which are additionally loop suppressed.

In the limit that conformal symmetry violating effects arising from the operator $\mathcal{O}$ are neglected, it follows from Eqs. (114) and (115) that if the gauge kinetic term in the low energy effective theory,

$$-\frac{1}{4}\frac{1}{g_{IR}^2}F_{\mu\nu}F^{\mu\nu} = -\frac{1}{4}\left\{\frac{1}{g_{UV}^2} + \frac{C}{8\pi^2}\right\}F_{\mu\nu}F^{\mu\nu} \ , \tag{117}$$

is to be invariant under infinitesimal scale transformations, the conformal compensator must couple as

$$\frac{(b_< - b_>)}{32\pi^2}\log\left(\frac{\chi}{f}\right)F_{\mu\nu}F^{\mu\nu} \tag{118}$$

in the Lagrangian. Expanding this out in terms of $\sigma(x)$, to leading order in inverse powers of f, we find for the dilaton coupling

$$\frac{(b_< - b_>)}{32\pi^2}\frac{\sigma}{f}F_{\mu\nu}F^{\mu\nu} \ . \tag{119}$$

This agrees with the result in the literature for the coupling of the radion to massless bulk gauge bosons in Randall-Sundrum models [45, 46]. Our analysis establishes that this result is valid beyond the large N limit. This formula is valid at scales slightly below the strong coupling scale $4\pi f$, and must be renormalization group evolved to the dilaton mass. If the conformal sector does not transform under the SM color group, as may be the case in theories where the top quark is not a composite of the strong dynamics, then $b_< = b_>$ and the gluon does not couple to the dilaton at this order. In such a scenario, the leading interaction of the dilaton with the gluons is through a loop of top quarks, just as for the Higgs in the SM.

When conformal symmetry violating effects arising from the operator $\mathcal{O}$ are included, this formula will receive corrections. The leading effect arises from the fact that the value of the gauge coupling at low energies depends on the detailed spectrum of states at the threshold, which in turn depends on $\hat{\lambda}_{\mathcal{O}}$. As a consequence the constant C in Eq. (116) is in general a function of $\hat{\lambda}_{\mathcal{O}}$. Now, conformal symmetry ensures that in the low energy effective theory C depends on $\overline{\lambda}_{\mathcal{O}}$ in the specific combination $C(\overline{\lambda}_{\mathcal{O}}\chi^{-\epsilon})$. Since the Lagrangian is limited to terms with positive integer powers of $\hat{\lambda}_{\mathcal{O}}$, we can expand C as

$$C = \left[C_0 + \sum_{n=1}^{\infty} C_n \overline{\lambda}_{\mathcal{O}}^n \, \chi^{(-n\epsilon)} \right] . \tag{120}$$

Inserting this into Eq. (117) and expanding in powers of σ, we find that the dilaton couplings take the form

$$\frac{\sigma}{f} \left[\frac{(b_< - b_>)}{32\pi^2} + \frac{\bar{c}_A}{32\pi^2} \epsilon \overline{\lambda}_{\mathcal{O}} f^{-\epsilon} \right] F_{\mu\nu}F^{\mu\nu} . \tag{121}$$

Here the dimensionless constant $\bar{c}_A$ is expected to be of order the number of states that transform under the gauge symmetry that have masses at the threshold, so that $\bar{c}_A \sim (b_< - b_>)$. We can therefore rewrite Eq. (121) as

$$\frac{\sigma}{f} \frac{(b_< - b_>)}{32\pi^2} \left[1 + c_A \epsilon \right] F_{\mu\nu}F^{\mu\nu} , \tag{122}$$

where the dimensionless constant c_A is of order $\overline{\lambda}_{\mathcal{O}} f^{-\epsilon}$, which is the value of $\hat{\lambda}_{\mathcal{O}}$ at the symmetry breaking scale f. We see from this that in this scenario, corrections to the dilaton couplings to massless gauge bosons arising from conformal symmetry violating effects are subleading, being suppressed not just by m_σ^2/Λ^2, but also by a loop factor. This is in contrast to the case of composite gauge bosons considered in the previous section.

B. Couplings to Fermions

1. Elementary Fermions

Next we consider the dilaton couplings to the SM fermions, which we label by Q, U^c, D^c, L and E^c. These depend on how the fermion masses are generated. One possibility is that the fermion masses arise from a contact term that couples a scalar operator $\mathcal{H}$ in the conformal field theory that carries the gauge quantum numbers of the SM Higgs to elementary fermions. For the up-type quarks, this takes the form

$$y^{ij} \mathcal{H} Q_i U_j^c + \text{ h.c.} \tag{123}$$

in the Lagrangian. Here i and j are flavor indices. This leads to a mass term

$$m^{ij} Q_i U_j^c + \text{ h.c.} \tag{124}$$

in the potential of the low energy effective theory. The generalization to the down-type quarks and leptons is straightforward. In the limit that y^{ij} is set to zero the ultraviolet theory has a $\mathrm{U}(3)_Q \times \mathrm{U}(3)_U$ flavor symmetry, which can be restored by promoting y^{ij} to a spurion that transforms as an anti-fundamental under each of these symmetries. By requiring that the low energy effective theory be invariant under this spurious flavor symmetry, it follows that m^{ij} is proportional to y^{ij} to lowest order in the couplings y.

It has been shown that the flavor problem and the large mass of the top quark can both be addressed in this framework if the operator $\mathcal{H}$ has dimension $\Delta_{\mathcal{H}} \lesssim 1.3$. However, if the hierarchy problem is to be solved, the dimension $\Delta_{\mathcal{H}^\dagger\mathcal{H}}$ of the operator $\mathcal{H}^\dagger\mathcal{H}$ must satisfy $\Delta_{\mathcal{H}^\dagger\mathcal{H}} \gtrsim 4$ [9]. Determining whether scalar operators that satisfy these criteria can exist in unitary, causal conformal field theories is an open question that has attracted considerable recent interest [55, 56]. Note that this condition cannot be satisfied in the large N limit, and therefore realistic models of this type cannot be constructed within the Randall-Sundrum framework.

In order to determine the coupling of the dilaton to the up-type quarks, we make the coupling in Eq. (123) formally invariant under scale transformations by promoting y^{ij} to a spurion that transforms as $y^{ij} \to y'^{ij} = e^{\omega(1-\Delta_{\mathcal{H}})} y^{ij}$ under $x \to x' = e^{-\omega}x$. Then the coupling of the dilaton to the up-type quarks in the effective theory must respect this symmetry. Since the quark mass matrix m^{ij} is proportional to y^{ij} the conformal compensator couples as

$$m^{ij} \left(\frac{\chi}{f} \right)^{\Delta_{\mathcal{H}}} Q_i U_j^c + \text{ h.c.} \tag{125}$$

Then to lowest order in inverse powers of f, the dilaton coupling to up-type quarks takes the form [28]

$$m^{ij} \Delta_{\mathcal{H}} \left(\frac{\sigma}{f} \right) Q_i U_j^c + \text{ h.c.} \tag{126}$$

We see that to the extent that $\Delta_{\mathcal{H}}$ differs from one, the dilaton couplings to fermions can differ significantly from those of a SM Higgs. Once effects of the operator $\mathcal{O}$ are included, Eq. (126) is modified to

$$m^{ij}\left(\Delta_{\mathcal{H}}+c_q\epsilon\right)\frac{\sigma}{f}Q_iU^c_j+\text{ h.c. }, \tag{127}$$

where the dimensionless parameter c_q is of order $\overline{\lambda}_{\mathcal{O}}f^{-\epsilon}$. We see that corrections to Eq. (126) from conformal symmetry violating effects are of order m^2_σ/Λ^2, and under control.

More generally, there could be several scalar operators $\mathcal{H}_\alpha$ in the conformal field theory that couple to the SM fermions. The coupling in Eq. (123) then generalizes to

$$y^{\alpha ij}\mathcal{H}_\alpha Q_iU^c_j+\text{ h.c. }, \tag{128}$$

where the index α runs over all the scalar operators in the theory with the quantum numbers of the SM Higgs. However, operators with dimension significantly larger than one are not expected to play a significant role.

It follows from the $\mathrm{U}(3)_Q\times$ $\mathrm{U}(3)_U$ flavor symmetry that the up-type fermion masses depend on the couplings $y^{\alpha ij}$ as

$$m^{ij}=y^{\alpha ij}D_\alpha\ , \tag{129}$$

where the parameters D_α depends on the details of the conformal field theory. In order to determine the couplings of the dilaton, we make the coupling in Eq. (128) formally invariant under scale transformations by promoting the $y^{\alpha ij}$ to spurions that transform as

$$y^{\alpha ij}\to y'^{\alpha ij}=e^{\omega\left(1-\Delta_{\mathcal{H}_{(\alpha)}}\right)}y^{(\alpha)ij}\ , \tag{130}$$

under $x\to x'=e^{-\omega}x$. There is no sum over α on the right hand side of this equation. The various terms in the sum on the right hand side of Eq. (129) transform differently under this transformation. In order to account for this we define

$$m^{\alpha ij}=y^{(\alpha)ij}D_{(\alpha)}\ , \tag{131}$$

where again there is no sum over α on the right hand side of this equation. Then the requirement that the fermion mass in the low energy effective theory be formally invariant under this symmetry constrains the conformal compensator to couple as

$$m^{\alpha ij}\left(\frac{\chi}{f}\right)^{\Delta_{\mathcal{H}_\alpha}}Q_iU^c_j+\text{ h.c.} \tag{132}$$

in the potential. This leads to the dilaton coupling

$$m^{\alpha ij}\Delta_{\mathcal{H}_\alpha}\frac{\sigma}{f}Q_iU^c_j+\text{ h.c.} \tag{133}$$

We see from this that if the $\Delta_{\mathcal{H}_\alpha}$ are not all equal, the couplings of the dilaton in the low energy effective theory violate flavor. However, in the absence of large cancellations among the contributions of different operators to the quark masses, the matrix $m^{\alpha ij}\Delta_{\mathcal{H}_\alpha}$ will be somewhat aligned with the quark mass matrix m^{ij}, leading to suppression of flavor violation.

2. Partially Composite Fermions

Another possible origin for the fermion masses is that the SM quarks and leptons are partial composites of the strongly interacting sector [57]. This scenario can arise if the theory contains elementary fermions Q_i, U^c_i, D^c_i, L_i and E^c_i with the same gauge quantum numbers as the corresponding SM fermions that mix with operators in the conformal field theory. The physical SM fermions emerge as a linear combination of the corresponding elementary particles and states associated with the strongly interacting sector. Within the Randall-Sundrum framework, this corresponds to putting the SM fermions in the bulk of the space [58].

To understand this in greater detail, let us consider the mass terms for the up-type quarks. These can be generated if the conformal field theory contains fermionic operators $\mathcal{Q}^c_\alpha$ and $\mathcal{U}_\alpha$, with dimensions $\Delta_{\mathcal{Q}}$ and $\Delta_{\mathcal{U}}$ respectively, that couple to elementary fermions Q_i and U^c_i in the Lagrangian as

$$y^{\alpha i}_{\mathcal{Q}}\mathcal{Q}^c_\alpha Q_i+y^{\beta j}_{\mathcal{U}}\mathcal{U}_\beta U^c_j+\text{ h.c.} \tag{134}$$

We assume that the indices α and β, which run from 1 to 3, are associated with an internal U(3) symmetry of the conformal sector so that $\Delta_{\mathcal{Q}}$ and $\Delta_{\mathcal{U}}$ are independent of α and β. We will relax this assumption later. If $\Delta_{\mathcal{Q}}$ and $\Delta_{\mathcal{U}}$ are close to 5/2, these interactions correspond to marginal operators in the conformal field theory. These couplings will generate up-type quark masses in the potential of the form

$$m^{ij}Q_iU^c_j+\text{ h.c.} \tag{135}$$

This framework can be extended to the down-type quarks and leptons in a straightforward way. The AdS/CFT correspondence relates the operator dimensions $\Delta_{\mathcal{Q}}$ and $\Delta_{\mathcal{U}}$ to the mass terms for bulk fermions in Randall-Sundrum models.

In the limit that the couplings $y_{\mathcal{Q}}$ and $y_{\mathcal{U}}$ are set to zero the ultraviolet theory has a $\mathrm{U}(3)_Q\times$ $\mathrm{U}(3)_U$ flavor symmetry. This symmetry can be restored by promoting $y_{\mathcal{Q}}$ and $y_{\mathcal{U}}$ to spurions that transform as anti-fundamentals under $\mathrm{U}(3)_Q$ and $\mathrm{U}(3)_U$ respectively. Then, requiring the low energy effective theory to respect this spurious symmetry constrains the mass matrix to be proportional to the product of $y_{\mathcal{U}}$ and $y_{\mathcal{Q}}$,

$$m^{ij}\propto\left[y^T_{\mathcal{Q}}\,y_{\mathcal{U}}\right]^{ij}\ , \tag{136}$$

to lowest order in the couplings y. The kinetic terms of the quarks in the low energy effective theory also receive corrections from the couplings $y_{\mathcal{Q}}$ and $y_{\mathcal{U}}$ of the form

$$\Delta Z_Q\overline{Q}\gamma^\mu D_\mu Q+\Delta Z_U\overline{U}^c\gamma^\mu D_\mu U^c \tag{137}$$

where

$$\begin{aligned}\Delta Z_Q\ &\sim\ \frac{1}{16\pi^2}\frac{y^\dagger_{\mathcal{Q}}y_{\mathcal{Q}}}{f^{5-2\Delta_{\mathcal{Q}}}}\\ \Delta Z_U\ &\sim\ \frac{1}{16\pi^2}\frac{y^\dagger_{\mathcal{U}}y_{\mathcal{U}}}{f^{5-2\Delta_{\mathcal{U}}}}\ .\end{aligned} \tag{138}$$

The corrections to the kinetic terms are a consequence of the fact that the fermions in the low energy theory are partially composite.

In order to determine the coupling of the dilaton to the up-type quarks, we promote $y_{\mathcal{Q}}$ and $y_{\mathcal{U}}$ to spurions that transform as $y_{\mathcal{Q}} \to y'_{\mathcal{Q}} = e^{\omega(5/2-\Delta_{\mathcal{Q}})} y_{\mathcal{Q}}$ and $y_{\mathcal{U}} \to y'_{\mathcal{U}} = e^{\omega(5/2-\Delta_{\mathcal{U}})} y_{\mathcal{U}}$ under $x \to x' = e^{-\omega} x$. Then the couplings (134) are formally invariant under scale transformations, and the conformal compensator couples to quarks so as to make low energy effective theory consistent with this symmetry. To lowest order in powers of $y_{\mathcal{Q}}$ and $y_{\mathcal{U}}$, and neglecting effects arising from $\mathcal{O}$, this coupling takes the form

$$m^{ij} Q_i U^c_j \left(\frac{\chi}{f}\right)^{(\Delta_{\mathcal{U}}+\Delta_{\mathcal{Q}}-4)} + \text{ h.c.} \tag{139}$$

in the potential. This leads to the dilaton couplings

$$m^{ij} \left(\Delta_{\mathcal{U}} + \Delta_{\mathcal{Q}} - 4\right) \frac{\sigma}{f} Q_i U^c_j + \text{ h.c.} \tag{140}$$

This agrees with the results in the literature for the coupling of the dilaton to partially composite fermions in the large N limit [28], and for the coupling of the radion to bulk fermions in the Randall-Sundrum model [45, 46]. Our analysis establishes that these results are valid beyond the large N limit.

When effects of the operator $\mathcal{O}$ are included, Eq. (140) is modified to

$$m^{ij} \left[\left(\Delta_{\mathcal{U}} + \Delta_{\mathcal{Q}} - 4\right) + c_q \epsilon\right] \frac{\sigma}{f} Q_i U^c_j + \text{ h.c.} \,, \tag{141}$$

where c_q is of order $\overline{\lambda}_{\mathcal{O}} f^{-\epsilon}$, which is the value of $\hat{\lambda}_{\mathcal{O}}$ at the scale f. In obtaining this expression, we have assumed that the operator $\mathcal{O}$ does not break the approximate SM flavor symmetries, or the internal U(3) symmetry of the conformal sector. It follows that corrections to Eq. (140) from conformal symmetry violating effects are suppressed by m_σ^2/Λ^2 and under good theoretical control.

There are additional contributions to the dilaton coupling to quarks associated with the corrections to the kinetic terms, Eq. (137). However, using the equations of motion, it can be shown these contributions are higher order in $y_{\mathcal{Q}}$ and $y_{\mathcal{U}}$ than the effects we have considered, and are therefore suppressed.

In the more general case the operators $\mathcal{Q}^c_\alpha$ and $\mathcal{U}_\alpha$, could have dimensions $\Delta_{\mathcal{Q}_\alpha}$ and $\Delta_{\mathcal{U}_\alpha}$ that depend on the flavor index α. Then it follows from the spurious flavor symmetries that the SM fermion masses depend on the couplings $y_{\mathcal{Q}}$ and $y_{\mathcal{U}}$ as

$$m^{ij} = y_{\mathcal{Q}}^{\alpha i} y_{\mathcal{U}}^{\beta j} D_{\alpha\beta} \,, \tag{142}$$

where the parameter $D_{\alpha\beta}$ depends on the details of the conformal field theory. We can make the theory formally invariant under scale transformations by promoting $y_{\mathcal{Q}}$ and $y_{\mathcal{U}}$ to spurions that transform as

$$\begin{aligned} y_{\mathcal{Q}}^{\alpha i} &\to y'^{\alpha i}_{\mathcal{Q}} = e^{\omega\left(5/2-\Delta_{\mathcal{Q}_{(\alpha)}}\right)} y_{\mathcal{Q}}^{(\alpha) i} \\ y_{\mathcal{U}}^{\alpha i} &\to y'^{\alpha i}_{\mathcal{U}} = e^{\omega\left(5/2-\Delta_{\mathcal{U}_{(\alpha)}}\right)} y_{\mathcal{U}}^{(\alpha) i} \end{aligned} \tag{143}$$

under $x \to x' = e^{-\omega} x^\mu$, where there is no sum over α on the right hand side of the equations. The couplings of the dilaton in the low energy effective theory must respect this symmetry. We define

$$m^{\alpha\beta ij} = y_{\mathcal{Q}}^{(\alpha)i} y_{\mathcal{U}}^{(\beta)j} D_{(\alpha)(\beta)} \,, \tag{144}$$

where again there is no sum over α and β on the right hand side of the equation. We also define

$$\Delta^{\mathcal{QU}}_{\alpha\beta} = \Delta_{\mathcal{Q}_\alpha} + \Delta_{\mathcal{U}_\beta} - 4 \,. \tag{145}$$

In terms of these new variables the coupling of the conformal compensator to the up-type quarks can be expressed as

$$m^{\alpha\beta ij} Q_i U^c_j \left(\frac{\chi}{f}\right)^{\Delta^{\mathcal{QU}}_{\alpha\beta}} + \text{ h.c.} \tag{146}$$

Expanding this out, we find for the dilaton couplings in the potential

$$m^{\alpha\beta ij} \Delta^{\mathcal{QU}}_{\alpha\beta} \frac{\sigma}{f} Q_i U^c_j + \text{ h.c.} \tag{147}$$

It follows that in this scenario, the couplings of the dilaton to the quarks in the low energy effective theory violate flavor. However, in the absence of large cancellations among the contributions of the different $y_{\mathcal{Q}}$ and $y_{\mathcal{U}}$ to m^{ij}, we expect some degree of alignment between the quark mass matrix and the dilaton coupling matrix, which may be sufficient to satisfy flavor bounds.

Since $\mathcal{Q}^c$ and $\mathcal{U}$ are part of the strongly interacting sector, they must arise from complete multiplets of $\text{SU}(2)_\text{L} \times \text{SU}(2)_\text{R}$. There are two distinct possibilities for the realization of this symmetry, which we consider in turn.

The first possibility is that $\mathcal{Q}^c$ transforms as $(2,1)$ under $\text{SU}(2)_\text{L} \times \text{SU}(2)_\text{R}$ while $\mathcal{U}$ is partnered by another state $\mathcal{D}$, and together they transform as a $(1,2)$. In the context of Randall-Sundrum models, this realization of custodial symmetry was first proposed in [59]. The large mass of the top quark implies that the couplings $y_{\mathcal{Q}}$ and $y_{\mathcal{U}}$ must be sizable for the third generation quarks. This realization leads to mild tension with precision electroweak tests, since $y_{\mathcal{U}}$ distinguishes between $\mathcal{U}$ and $\mathcal{D}$, and therefore violates custodial SU(2) symmetry.

The alternative possibility [60] is that $\mathcal{Q}^c$ is partnered with a new state $\hat{\mathcal{Q}}^c$, and together they transform as $(2,2)$ under $\text{SU}(2)_\text{L} \times \text{SU}(2)_\text{R}$. Meanwhile, $\mathcal{U}$ is now just a singlet. In this realization of the extended symmetry, it is $y_{\mathcal{Q}}$ that violates custodial SU(2) and leads to tension with precision tests. This difficulty can be avoided if the third generation SU(2) singlet up-type quark U^c_3 is a composite of the strongly interacting sector. This allows $y_{\mathcal{Q}}$ to remain small enough to avoid conflict with the bound. In this scenario Eq. (147) remains valid, the only difference being that $\Delta_{\mathcal{U}_3}$ now takes the value 5/2.

V. HIGGS AS A PNGB

Next we consider theories where the SM Higgs doublet emerges as the pNGB associated with the breaking of an approximate global symmetry by strong conformal dynamics. For concreteness, we will take the global symmetry to be SO(6), which is broken to SO(5). An SU(2)× U(1) subgroup of the unbroken SO(5) is gauged, and identified with the electroweak gauge sector of the SM. Of the 5 pNGBs, 4 are identified with the SM Higgs doublet, while the remaining one is a SM singlet.

For the purpose of writing interactions, it is convenient to work in a framework where we keep only the symmetries associated with the SU(3)× U(1) subgroup of the non-linearly realized SO(6) global symmetry manifest. As shown in [61], the 5 NGBs associated with the breaking of SO(6) to SO(5) can be identified with the 5 NGBs arising from the breaking of the SU(3)× U(1) subgroup of SO(6) to SU(2)× U(1), since the corresponding coset spaces are identical. We parametrize the NGBs as h^a, where a runs from 1 to 5. Rather than work with the h^a directly it is more convenient to construct an object ϕ which transforms linearly under SU(3)× U(1).

$$\phi = \hat{f} \exp\left(i h^a t^a\right) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} . \tag{148}$$

Note that we are employing a convention where the h^a carry no mass dimension. We expect that $\hat{f}$ and f will be of the same order, since the same dynamics is responsible for the breaking of both conformal symmetry and the global symmetry. The 5 matrices t^a span [SU(3)× U(1)/SU(2)× U(1)], and are chosen as

$$\{t^a\} = \left\{ \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \right\}. \tag{149}$$

This choice allows us to take h^a, $a = 1 \to 4$ to represent the SM Higgs doublet, which we denote by h, while h^5 represents the additional singlet.

The low energy effective Lagrangian will in general contain all possible operators consistent with the SU(3)× U(1) global symmetry, but with restrictions on the coefficients of various terms enforced by the larger SO(6) symmetry. In particular the dangerous custodial SU(2) violating operator

$$\left|\phi^\dagger D_\mu \phi - (D_\mu \phi)^\dagger \phi\right|^2 , \tag{150}$$

while allowed by SU(3)× U(1), is forbidden by SO(6). Here D_μ is the gauge covariant derivative with respect to the SM SU(2) × U(1) gauge symmetry.

The requirement of scale invariance implies that the non-linear sigma model condition $|\phi|^2 = \hat{f}^2$ becomes $|\phi|^2 = \hat{f}^2 \hat{\chi}^2$. This means that we can make the low energy effective theory for the pNGBs invariant under scale transformations, up to terms arising from effects that explicitly violate the conformal and global symmetries, by making the replacement $\hat{f} \to \hat{f}\hat{\chi}$ in Eq. (148). The net effect is that in the low energy effective theory the h^a transform as fields with scaling dimension equal to zero, up to effects that violate conformal symmetry. This allows us to determine the form of the dilaton couplings to the SM fields. A major simplification is that since h has no scaling dimension, when replaced by its VEV the various operators have exactly the same scaling dimensions as in the technicolor models of the previous section, and many results can simply be carried over.

A. Couplings to Gauge Bosons

We begin by considering the dilaton couplings to the weak gauge bosons of the SM. These arise from the gauge covariant kinetic term for ϕ,

$$\left(D_\mu \phi\right)^\dagger D^\mu \phi . \tag{151}$$

Expanding out ϕ to lowest order in h, we obtain the gauge covariant kinetic term for the SM Higgs doublet

$$\hat{\chi}^2 \hat{f}^2 \left(D_\mu h\right)^\dagger D^\mu h , \tag{152}$$

Working in unitary gauge, and replacing h by its VEV, we find the coupling of the conformal compensator to the W bosons in the Lagrangian

$$\frac{m_W^2}{g^2} \frac{\chi^2}{f^2} W_\mu^+ W^{\mu\,-} . \tag{153}$$

This leads to the dilaton coupling

$$2 \frac{\sigma}{f} \frac{m_W^2}{g^2} W_\mu^+ W^{\mu\,-} . \tag{154}$$

As expected, this is identical to the corresponding formula in the technicolor case. When conformal symmetry violating effects arising from $\mathcal{O}$ are incorporated, the non-linear sigma model condition is modified to

$$|\phi|^2 = \hat{f}^2 \hat{\chi}^2 \left[1 + \sum_{n=1}^{\infty} \alpha_{\phi,n} \overline{\lambda}_{\mathcal{O}}^n \, \chi^{(-n\epsilon)}\right] , \tag{155}$$

where the dimensionless parameters $\alpha_{\phi,n}$ are expected to be of order one. Then the dilaton coupling to W bosons is modified to

$$\frac{\sigma}{f} \frac{m_W^2}{g^2} \left(2 + c_W \epsilon\right) W_\mu^+ W^{\mu\,-} , \tag{156}$$

where c_W is of order $\overline{\lambda}_{\mathcal{O}} f^{-\epsilon}$. We see that the corrections are suppressed by m_σ^2/Λ^2, exactly as in the technicolor case.

Next we consider dilaton couplings to the massless gauge bosons of the SM, the gluons and the photon. The leading effect which breaks conformal invariance is again the running of the gauge couplings, just as in the previous section. The results can simply be carried over, and are given by Eq. (122),

$$\frac{\sigma}{f}\frac{(b_{<}-b_{>})}{32\pi^2}\left[1+c_A\epsilon\right]F_{\mu\nu}F^{\mu\nu}\ . \tag{157}$$

As can be seen from this formula, corrections to the dilaton couplings from conformal symmetry breaking effects are suppressed by m_σ^2/Λ^2 and also by a loop factor, and are generally small.

B. Couplings to Fermions

1. Elementary Fermions

Next we consider dilaton couplings to the SM fermions. We begin with the case where the SM fermions are elementary, and their masses arise from direct contact interactions with operators in the conformal field theory. The up-type fermion masses arise from terms in the Lagrangian of the form

$$\hat{y}^{ij}\mathcal{H}Q_iU_j^c+\ \text{h.c.} \tag{158}$$

that break the global symmetry. Here the operator $\mathcal{H}$ has the quantum numbers of the SM Higgs doublet. This leads to Yukawa couplings for the up-type quarks in the potential of the low energy effective theory,

$$y^{ij}\left(\hat{f}h\right)Q_iU_j^c+\ \text{h.c.}\ , \tag{159}$$

where we are neglecting higher order terms in h which may also arise from the term in Eq. (158). It follows from the flavor symmetries that y^{ij} is proportional to $\hat{y}^{ij}$ to lowest order in the couplings $\hat{y}$. We can find the dilaton couplings by promoting $\hat{y}^{ij}$ to a spurion exactly as in the technicolor case. Noting that h has no scaling dimension, it follows that the conformal compensator couples to up-type quarks as

$$y^{ij}\left(\frac{\chi}{f}\right)^{\Delta_\mathcal{H}}\left(\hat{f}h\right)Q_iU_j^c+\ \text{h.c.} \tag{160}$$

in the potential. This leads to the dilaton coupling

$$y^{ij}\Delta_\mathcal{H}\frac{\sigma}{f}\left(\hat{f}h\right)Q_iU_j^c+\ \text{h.c.}\ . \tag{161}$$

Replacing h by its VEV we obtain

$$m^{ij}\Delta_\mathcal{H}\frac{\sigma}{f}Q_iU_j^c+\ \text{h.c.}\ , \tag{162}$$

exactly as in the technicolor case. When effects of the operator $\mathcal{O}$ are included, this again becomes

$$m^{ij}\left(\Delta_\mathcal{H}+c_q\epsilon\right)\frac{\sigma}{f}Q_iU_j^c\ , \tag{163}$$

where c_q is of order $\overline{\lambda}_\mathcal{O}f^{-\epsilon}$.

In the case where there are multiple operators $\mathcal{H}_\alpha$ that couple to the SM fermions,

$$\hat{y}_U^{\alpha ij}\mathcal{H}_\alpha Q_iU_j^c+\ \text{h.c.}\ , \tag{164}$$

Eq. (162) generalizes to the corresponding formula in the technicolor case, Eq. (133).

2. Partially Composite Fermions

We move on to the case where the SM quarks and leptons are partial composites of the strongly interacting sector. We introduce elementary fermions Q_i, U_i^c, D_i^c, L_i and E_i^c that have the same gauge quantum numbers as the corresponding SM fermions, and which mix with operators in the conformal field theory. The observed SM fermions are linear combinations of the corresponding elementary particles and states associated with the strongly interacting sector.

Mass terms for the up-type quarks arise from couplings of fermionic operators $\mathcal{Q}_\alpha^c$ and $\mathcal{U}_\alpha$, with dimensions $\Delta_\mathcal{Q}$ and $\Delta_\mathcal{U}$ respectively, to the elementary fermions Q_i and U_i^c in the Lagrangian,

$$y_\mathcal{Q}^{\alpha i}\mathcal{Q}_\alpha^cQ_i+y_\mathcal{U}^{\beta i}\mathcal{U}_\beta U_i^c+\ \text{h.c.} \tag{165}$$

We assume that the indices α and β, which run from 1 to 3, are associated with an internal U(3) symmetry of the conformal sector so that $\Delta_\mathcal{Q}$ and $\Delta_\mathcal{U}$ are independent of α and β. We will relax this assumption later. We can determine the coupling of the dilaton to the up-type quarks by promoting $y_\mathcal{Q}$ and $y_\mathcal{U}$ to spurions, exactly as in the technicolor case. Noting that h has scaling dimension zero, we find that the conformal compensator couples as

$$y^{ij}\left(\frac{\chi}{f}\right)^{(\Delta_\mathcal{U}+\Delta_\mathcal{Q}-4)}\left(\hat{f}h\right)Q_iU_j^c+\ \text{h.c.} \tag{166}$$

Replacing h by its VEV and expanding χ out in terms of σ, we obtain

$$m^{ij}\left(\Delta_\mathcal{U}+\Delta_\mathcal{Q}-4\right)\frac{\sigma}{f}Q_iU_j^c+\ \text{h.c.}\ , \tag{167}$$

which is identical to the corresponding formula in the technicolor case, Eq. (140). When effects of the operator $\mathcal{O}$ are included, Eq. (167) receives corrections, and is again modified to

$$m_U^{ij}\left[\left(\Delta_\mathcal{U}+\Delta_\mathcal{Q}-4\right)+c_q\epsilon\right]\frac{\sigma}{f}Q_iU_j^c+\ \text{h.c.} \tag{168}$$

The corrections are suppressed by m_σ^2/Λ^2, and therefore small. In the more general case where the operators $\mathcal{Q}_\alpha^c$ and $\mathcal{U}_\alpha$ have dimensions $\Delta_{\mathcal{Q}_\alpha}$ and $\Delta_{\mathcal{U}_\alpha}$ that depend on the index α, Eq. (167) generalizes to the corresponding formula in the technicolor case, Eq. (147).

Since $\mathcal{Q}$ and $\mathcal{U}$ are part of the strongly interacting sector, they must arise from complete multiplets of O(6).

Perhaps the simplest possibility is that $\mathcal{Q}^c$ constitutes part of a multiplet that transforms as a fundamental of O(6), while $\mathcal{U}$ is just a singlet. In this realization of the extended symmetry $y_{\mathcal{Q}}$ violates custodial SU(2). The large mass of the top quark means that this coupling must be large for the third generation, leading to tension with precision tests. This difficulty can be avoided if the third generation SU(2) singlet up-type quark U_3^c is a composite of the strongly interacting sector. This allows $y_{\mathcal{Q}}$ to remain small enough to avoid conflict with the bound. In this scenario Eq. (147) remains valid, but with $\Delta_{\mathcal{U}_3}$ taking the value 5/2.

C. Coupling to the Higgs

Finally we consider the dilaton coupling to the SM Higgs. In general, this receives contributions from both the Higgs kinetic term and the Higgs potential. From the kinetic term for the Higgs doublet, Eq. (152), we obtain the coupling

$$\frac{\sigma}{f}\partial_\mu\rho\partial^\mu\rho \tag{169}$$

in the Lagrangian. Here ρ is the canonically normalized SM Higgs field, and we are working only to quadratic order in ρ. When corrections arising from the symmetry violating parameter $\mathcal{O}$ are included, this is modified to

$$\frac{\sigma}{f}\left[1+c_H\epsilon\right]\partial_\mu\rho\partial^\mu\rho \; . \tag{170}$$

where c_H is of order $\overline{\lambda}_{\mathcal{O}} f^{-\epsilon}$.

The kinetic term for ϕ, Eq. (151), does not lead to mixing between between the dilaton and the SM Higgs field. Other two derivative terms, such as

$$\frac{\partial^\mu\chi}{\chi}\left[\phi^\dagger D_\mu\phi+(D_\mu\phi)^\dagger\phi\right] \; , \tag{171}$$

also do not generate such mixing. This conclusion remains true when conformal symmetry violating effects are included.

In this scenario, the potential for the Higgs doublet can only arise from effects that explicitly violate the global symmetry, such as the SM gauge and Yukawa interactions. If all such effects, however, respect conformal symmetry, then the potential for the Higgs doublet is of the very restrictive form

$$V=\chi^4 V_0(h) \; . \tag{172}$$

A potential of this form does not lead to mixing between the SM Higgs and the dilaton after minimization. The reason is that when the Higgs field is expanded about its VEV, there is no linear term in ρ at the minimum of the potential $V(h)$. However, expanding $V(h)$ to quadratic order in ρ, we find a coupling of the dilaton to the Higgs of the form

$$2\frac{\sigma}{f}m_\rho^2\rho^2 \tag{173}$$

in the potential. This formula will receive corrections from any contribution to the Higgs potential that arises from an effect that violates conformal symmetry. Mixing between the Higgs and the dilaton may be generated by such effects.

In particular, when effects arising from the operator $\mathcal{O}$ is taken into account, the Higgs potential takes the more general form

$$V=\chi^4 V_0(h)+\sum_{n=1}^{\infty}\overline{\lambda}_{\mathcal{O}}^n\,\chi^{(4-n\epsilon)}V_n(h) \; . \tag{174}$$

Eq. (173) is consequently modified to

$$\frac{\sigma}{f}\left(2+c_\rho\epsilon\right)m_\rho^2\rho^2 \; , \tag{175}$$

where, if the symmetry violating terms contribute significantly to the potential so that at the minimum $V_1(h)$ is of order $V_0(h)$, c_ρ is expected to be of order $\overline{\lambda}_{\mathcal{O}} f^{-\epsilon}$. We see that the corrections are suppressed by m_σ^2/Λ^2. This effect also gives rise to mixing between the Higgs and the dilaton. However the mixing angle θ is small,

$$\theta \lesssim \epsilon\overline{\lambda}_{\mathcal{O}} f^{-\epsilon}\left(\frac{v}{f}\right)\sim\frac{m_\sigma^2}{\Lambda^2}\left(\frac{v}{f}\right) \; . \tag{176}$$

Here v is the electroweak VEV.

Since the SM gauge interactions also constitute an explicit breaking of conformal symmetry, there will be additional radiative corrections to the Higgs potential that are not of the simple form of Eq. (172). However, because the gauge interactions respect conformal symmetry at the classical level, and only break it through quantum effects, deviations away from this form are further loop suppressed, and generally small.

In theories where the top quarks are elementary or partially composite, the top Yukawa coupling also violates conformal symmetry. Then, if contributions to the Higgs potential from loops involving the top Yukawa coupling are sizable, there can be significant deviations away from the form of Eq. (172). We parametrize the coupling of the conformal compensator to the top quark as

$$\frac{m_t}{v}\left(\frac{\chi}{f}\right)^{(1+\bar{\Delta})}\left(\hat{f}h\right)\bar{t}t \; , \tag{177}$$

where $\bar{\Delta}$ is equal to zero if the top quarks are composite, is equal to $(\Delta_{\mathcal{H}}-1)$ if the top quarks are elementary, and is equal to $(\Delta_{\mathcal{U}_3}+\Delta_{\mathcal{Q}_3}-5)$ if the top quarks are partially composite. Then one loop corrections to the Higgs potential from the top loop, which we label by δV_t, are of the form

$$\chi^4\left[\sum_{n=1}^{2}\frac{\hat{\alpha}_{t,n}}{(16\pi^2)^{n-1}}\,\left(\frac{m_t}{v}\right)^{2n}\left(\frac{\chi}{f}\right)^{2n\bar{\Delta}}|h|^{2n}\right] \; . \tag{178}$$

Here the dimensionless parameters $\hat{\alpha}_{t,n}$ are of order one. Then Eq. (173) is modified to

$$\frac{\sigma}{f}\left[2+\bar{c}_\rho\bar{\Delta}\right]m_\rho^2\rho^2 \; . \tag{179}$$

If contributions to the Higgs potential arising from loops involving the top Yukawa coupling are significant, so that δV_t is comparable to V in Eq. (172) at the minimum, we expect $\bar{c}_\rho$ to be of order one. This effect also gives rise to mixing between the dilaton and the Higgs, but the mixing angle $\theta \lesssim \bar{\Delta} v/f$ is expected to be small in realistic models. Mixing will correct the dilaton couplings to other SM fields as well, and so a precise determination of these interactions requires this effect to be taken into account. We leave this for future work.

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[1] S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B [arXiv:1207.7235 [hep-ex]].
[2] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B [arXiv:1207.7214 [hep-ex]].
[3] L. Susskind, Phys. Rev. D **20**, 2619 (1979).
[4] S. Weinberg, Phys. Rev. D **13**, 974 (1976); Phys. Rev. D **19**, 1277 (1979).
[5] C. T. Hill and E. H. Simmons, Phys. Rept. **381**, 235 (2003) [Erratum-ibid. **390**, 553 (2004)] [hep-ph/0203079].
[6] B. Holdom and J. Terning, Phys. Lett. B **247**, 88 (1990).
[7] M. Golden and L. Randall, Nucl. Phys. B **361**, 3 (1991).
[8] M. E. Peskin and T. Takeuchi, Phys. Rev. D **46**, 381 (1992).
[9] M. A. Luty and T. Okui, JHEP **0609**, 070 (2006) [hep-ph/0409274].
[10] B. Holdom, Phys. Lett. B **150**, 301 (1985).
[11] T. W. Appelquist, D. Karabali and L. C. R. Wijewardhana, Phys. Rev. Lett. **57**, 957 (1986).
[12] K. Yamawaki, M. Bando and K. -i. Matumoto, Phys. Rev. Lett. **56**, 1335 (1986).
[13] T. Appelquist and L. C. R. Wijewardhana, Phys. Rev. D **35**, 774 (1987); Phys. Rev. D **36**, 568 (1987).
[14] H. Georgi and A. Pais, Phys. Rev. D **10**, 539 (1974); Phys. Rev. D **12**, 508 (1975).
[15] D. B. Kaplan and H. Georgi, Phys. Lett. B **136**, 183 (1984).
[16] D. B. Kaplan, H. Georgi and S. Dimopoulos, Phys. Lett. B **136**, 187 (1984); H. Georgi and D. B. Kaplan, Phys. Lett. B **145**, 216 (1984).
[17] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B **513**, 232 (2001) [hep-ph/0105239].
[18] N. Arkani-Hamed, A. G. Cohen, E. Katz and A. E. Nelson, JHEP **0207**, 034 (2002) [hep-ph/0206021];
[19] T. Gregoire and J. G. Wacker, JHEP **0208**, 019 (2002) [hep-ph/0206023]; I. Low, W. Skiba and D. Tucker-Smith, Phys. Rev. D **66**, 072001 (2002) [hep-ph/0207243]; D. E. Kaplan and M. Schmaltz, JHEP **0310**, 039 (2003) [hep-ph/0302049]. S. Chang and J. G. Wacker, Phys. Rev. D **69**, 035002 (2004) [hep-ph/0303001].
[20] Z. Chacko, H. -S. Goh and R. Harnik, Phys. Rev. Lett. **96**, 231802 (2006) [hep-ph/0506256]; R. Barbieri, T. Gregoire and L. J. Hall, hep-ph/0509242; Z. Chacko, Y. Nomura, M. Papucci and G. Perez, JHEP **0601**, 126 (2006) [hep-ph/0510273].
[21] Z. Chacko, H. -S. Goh and R. Harnik, JHEP **0601**, 108 (2006) [hep-ph/0512088]; H. -S. Goh and C. A. Krenke, Phys. Rev. D **76**, 115018 (2007) [arXiv:0707.3650 [hep-ph]].
[22] A. Salam and J. A. Strathdee, Phys. Rev. **184**, 1760 (1969); C. J. Isham, A. Salam and J. A. Strathdee, Phys. Lett. B **31**, 300 (1970).
[23] C. J. Isham, A. Salam and J. A. Strathdee, Annals Phys. **62**, 98 (1971).
[24] B. Zumino, in 1970 Brandeis Summer School, 'Lectures on Elementary Particles and Quantum Field Theory', edited by S. Deser *et. al.*, MIT Press (1970)
[25] J. R. Ellis, Nucl. Phys. B **22**, 478 (1970); Nucl. Phys. B **26**, 536 (1971).
[26] R. Rattazzi and A. Zaffaroni, JHEP **0104**, 021 (2001) [hep-th/0012248].
[27] W. D. Goldberger, B. Grinstein and W. Skiba, Phys. Rev. Lett. **100**, 111802 (2008) [arXiv:0708.1463 [hep-ph]].
[28] L. Vecchi, Phys. Rev. D **82**, 076009 (2010) [arXiv:1002.1721 [hep-ph]].
[29] R. Rattazzi, "The Naturally Light Dilaton," *Talk at Planck 2010*.
[30] T. Appelquist and Y. Bai, Phys. Rev. D **82**, 071701 (2010) [arXiv:1006.4375 [hep-ph]].
[31] K. Cheung and T. -C. Yuan, Phys. Rev. Lett. **108**, 141602 (2012) [arXiv:1112.4146 [hep-ph]].
[32] S. Matsuzaki and K. Yamawaki, Phys. Rev. D **85**, 095020 (2012) [arXiv:1201.4722 [hep-ph]]; arXiv:1206.6703 [hep-ph]; arXiv:1207.5911 [hep-ph]; arXiv:1209.2017 [hep-ph].
[33] B. Grzadkowski, J. F. Gunion and M. Toharia, Phys. Lett. B **712**, 70 (2012) [arXiv:1202.5017 [hep-ph]].
[34] D. Elander and M. Piai, arXiv:1208.0546 [hep-ph].
[35] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [hep-th/9711200].
[36] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998) [hep-th/9802150].
[37] H. L. Verlinde, Nucl. Phys. B **580**, 264 (2000) [hep-th/9906182].
[38] E. P. Verlinde and H. L. Verlinde, JHEP **0005**, 034 (2000) [hep-th/9912018].
[39] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [hep-ph/9905221].
[40] C. Csaki, C. Grojean, L. Pilo and J. Terning, Phys. Rev. Lett. **92**, 101802 (2004) [hep-ph/0308038].
[41] R. Contino, Y. Nomura and A. Pomarol, Nucl. Phys. B **671**, 148 (2003) [hep-ph/0306259]; K. Agashe, R. Contino and A. Pomarol, Nucl. Phys. B **719**, 165 (2005) [hep-ph/0412089].
[42] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83**, 4922 (1999) [hep-ph/9907447]; Phys. Lett. B **475**, 275 (2000) [hep-ph/9911457].
[43] G. F. Giudice, R. Rattazzi and J. D. Wells, Nucl. Phys. B **595**, 250 (2001) [hep-ph/0002178].

[44] C. Csaki, M. L. Graesser and G. D. Kribs, Phys. Rev. D **63**, 065002 (2001) [hep-th/0008151].
[45] T. G. Rizzo, JHEP **0206**, 056 (2002) [hep-ph/0205242].
[46] C. Csaki, J. Hubisz and S. J. Lee, Phys. Rev. D **76**, 125015 (2007) [arXiv:0705.3844 [hep-ph]].
[47] J. Polchinski, Nucl. Phys. B **303**, 226 (1988).
[48] J. -F. Fortin, B. Grinstein and A. Stergiou, Phys. Lett. B **704**, 74 (2011) [arXiv:1106.2540 [hep-th]]; arXiv:1107.3840 [hep-th]; arXiv:1202.4757 [hep-th]; arXiv:1206.2921 [hep-th]; arXiv:1208.3674 [hep-th].
[49] M. A. Luty, J. Polchinski and R. Rattazzi, arXiv:1204.5221 [hep-th].
[50] R. Sundrum, hep-th/0312212.
[51] J. Fan, W. D. Goldberger, A. Ross and W. Skiba, Phys. Rev. D **79**, 035017 (2009) [arXiv:0803.2040 [hep-ph]].
[52] V. Barger, M. Ishida and W. -Y. Keung, Phys. Rev. D **85**, 015024 (2012) [arXiv:1111.2580 [hep-ph]]; Phys. Rev. Lett. **108**, 101802 (2012) [arXiv:1111.4473 [hep-ph]].
[53] B. Coleppa, T. Gregoire and H. E. Logan, Phys. Rev. D **85**, 055001 (2012) [arXiv:1111.3276 [hep-ph]].
[54] B. A. Campbell, J. Ellis and K. A. Olive, JHEP **1203**, 026 (2012) [arXiv:1111.4495 [hep-ph]].
[55] R. Rattazzi, V. S. Rychkov, E. Tonni and A. Vichi, JHEP **0812**, 031 (2008) [arXiv:0807.0004 [hep-th]]; V. S. Rychkov and A. Vichi, Phys. Rev. D **80**, 045006 (2009) [arXiv:0905.2211 [hep-th]]; R. Rattazzi, S. Rychkov and A. Vichi, J. Phys. A A **44**, 035402 (2011) [arXiv:1009.5985 [hep-th]].
[56] D. Poland and D. Simmons-Duffin, JHEP **1105**, 017 (2011) [arXiv:1009.2087 [hep-th]].
[57] D. B. Kaplan, Nucl. Phys. B **365**, 259 (1991).
[58] Y. Grossman and M. Neubert, Phys. Lett. B **474**, 361 (2000) [hep-ph/9912408]; T. Gherghetta and A. Pomarol, Nucl. Phys. B **586**, 141 (2000) [hep-ph/0003129].
[59] K. Agashe, A. Delgado, M. J. May and R. Sundrum, JHEP **0308**, 050 (2003) [hep-ph/0308036].
[60] K. Agashe, R. Contino, L. Da Rold and A. Pomarol, Phys. Lett. B **641**, 62 (2006) [hep-ph/0605341].
[61] P. Batra and Z. Chacko, Phys. Rev. D **77**, 055015 (2008) [arXiv:0710.0333 [hep-ph]].